

# Early Termination of Small Loans in the Multifamily Mortgage Market<sup>1</sup>

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## **Executive Summary**

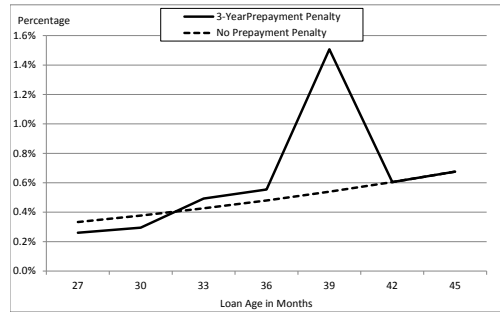
This report provides analysis and evaluation on early mortgage termination in the Fannie Mae multifamily loan pool. The objective is to identify those factors that uniquely explain prepayment and default in the “small loan” subset of mortgages in the Fannie Mae portfolio over the period 2005 through 2011. In the analysis that follows we consider contractual components, property performance, and both micro and macroeconomic factors and their ability to influence mortgage returns. The data utilized in the modeling includes mortgage loan records, local market capitalization and rental rates, state level unemployment, prevailing interest rates and property characteristics into the analysis.

In general, we find that the mortgage market functions as expected and that both borrowers and lenders react to the incentives provided by local market conditions, the property’s performance, and the specific contract provisions in the mortgage. The results also clarify the findings previously presented in an academic paper by Archer et al (2002) that loan to value ratios (LTV) and debt coverage ratios (DCRs) at origination are endogenous and not predictive of default or prepayment in a meaningful way. We show that using contemporaneous LTV and contemporaneous DCR breaks the endogeneity. Both the contemporaneous cash flow position (DCR) and equity position (LTV) have meaningful effects on both default and prepayment of multifamily loans.

Nearly all observed loans have balloon payments, prepayment penalties and/or yield maintenance contract provisions. The typical yield maintenance agreement requires the borrower to replace the expected mortgage payments with like-kind US Treasury expected payments. Key issues the literature has not addressed are whether and how prepayment suppression mechanisms affect default probabilities, and whether commonly accepted financial incentives to prepay and default still work when prepayment penalties and yield maintenance agreements are in effect. These provisions have profound impacts on the expected duration of the loans and the amount of credit risk. Both yield maintenance and prepayment penalties are largely successful in suppressing prepayments. When these provisions expire, prepayments increase dramatically, often by a factor of 5 (see adjoining illustration for example). The main

benefit of these provisions is that the expected duration of a pool of loans with similar expiration dates is fairly simple to calculate.

The cost of these provisions is that they tend to increase the probability of default over the whole life of the loans, and these probabilities tend to spike up around or slightly before the expiration date of the restrictions.



Balloon payments also influence prepayment by concentrating terminations around a single point in time (the balloon payment due date). Simulations show that for a 10-year term loan with a balloon payment at term pay-offs rise to 45 percent of remaining loans and defaults spike to almost 5 percent of remaining loans. These probabilities imply that prepayments cluster around the balloon date, as anticipated, but there is also increased default risk .

In summary, the detailed contract provisions of a commercial property mortgage are extremely important. Not knowing the details of the prepayment suppression provisions and the extent of amortizing will lead to gross miscalculations of expected mortgage yields and cash flows. In addition, for these small loans it is more important to know what is going on at the property than what is happening in the market overall. The contemporaneous market value of the property, debt service, and income and cash flow generated by the property are all key determinants in avoiding defaults and understanding when prepayment will occur.

## **Early Termination of Small Loans in the Multifamily Mortgage Market**

### **Abstract**

This paper uses micro level data on small (as defined by Fannie Mae) multifamily loans in the Fannie Mae loan portfolio to examine recent prepayment and default performance. A multinomial logit model is specified and estimated for cross sectional and time series mortgage data. The results document the independent statistical significance of contemporaneous payment/income and loan/ value ratios and unemployment rates as well as more commonly studied determinants of default such as loan age and the original loan/value ratio. Prepayment is closely linked to the expiration date of prepayment penalties and yield maintenance provisions. However, such contractual provisions also tend to create increases in credit risk. When a balloon payment is required, a prepayment penalty provision expires, or a yield maintenance provision expires, the probability of default increases approximately 5 times.

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## Introduction

One of the most widely studied topics in the mortgage finance literature is the early termination of loans through borrower default and prepayment. Much of the prior work is focused on the residential mortgage market. Although there are some studies that examine factors contributing to the early termination of commercial loans, very few of these target multifamily loans and none focus on smaller multifamily loans. This is an important void in the literature given the level of lending and securitization activity that both Fannie Mae and Freddie Mac conduct in this segment of the market. Loan purchase and securitization by Fannie Mae, Freddie Mac and private-label commercial mortgage-backed securities (CMBS) grew rapidly during the 1990s and accounted for more than one-half of the net growth in multifamily debt over the decade (Nothafft and Freund, 2003). Recently, multifamily issuance accounted for approximately 62 percent of all conduit and government sponsored enterprise security issuance in 2011, compared to approximately 21 percent in 2006 (Heschmeyer, 2012). According to the Mortgage Bankers Association 2009 Survey on Multifamily Lending, small loans comprised approximately 27% of the total multifamily market by dollar volume and 81% by number of loans.

Studies have approached the question of commercial (including multifamily and other property types) default and prepayment from the option theoretic perspective. Ciochetti et al (2003) find that an option framework is potentially useful for explaining early termination of commercial mortgages, relying on contemporaneous Loan to Value (LTV) and Debt Coverage Ratios (DCR) as proxies for the value of the option to terminate the loan. But the evidence is not consistent. For example, Archer et al (2002) find no evidence that initial LTV or the equity in the property has any impact on loan termination and some weak evidence that the property income relative to the debt service requirement (the measure of DCR) affects terminations. The authors hypothesize that initial LTV is ineffective in explaining early termination of commercial mortgages because it is endogenous to the terms of a mortgage, reflecting the lender's overall perception of risk in evaluating the potential loan. For example, lenders may require more equity when the property is viewed as more risky. If this type of tradeoff dominates underwriting, empirical results could even show that more equity is associated with

more defaults. On the other hand, it is more difficult for a lender to require more income, as compared with more equity, to cover the debt service. It is not surprising, then, that Archer et al (2002) find better empirical estimates for DCR than LTV. Ambrose and Sanders (2003) also fail to find a link between LTV and prepayment or default. They do find a strong relationship between the yield curve and mortgage termination.

In addition to examining the role of LTV and DCR, we estimate the effects of property characteristics, loan terms, and economic factors on the incidence of prepayment and default. Prepayment events are closely tied to prepayment penalties or yield maintenance provisions. Both are very effective in suppressing prepayments when the provisions are in effect, and there is the expected large spike in prepayments as the provisions expire. However, these mechanisms, used to make prepayment more predictable for the lender/investor, come with a cost. Near the expiration date (for both prepayment and yield maintenance provisions) the probability of default increases approximately 5 times. Loans with a balloon feature (not fully amortizing and require a balloon payment at the end of the loan's life) also experience substantial increases in default risk as the loan approaches maturity.

Our general findings suggest that property financial performance and terms of the loan, coupled with LTV and DCR, are the most important factors explaining early termination. This is in contrast to some earlier findings (such as Archer et al, 2002). Economic controls round out the variables, but often present inconsistent results. The extent to which our results can be generalized beyond the sample is uncertain, but our findings contribute to the limited collection of research on the determinants of multifamily mortgage termination.

In the following sections we provide a summary of the mortgage termination literature and discuss how it relates to multifamily loans; describe the empirical approach and the data; and present the results and our conclusions.

## **Motivation and Literature**

Recent reports indicate that as many as 10.3 percent of borrowers with a mortgage, or 8.8 million households, have been “upside-down” (in negative equity) on their mortgages (Leland, 2008), while the 60 day delinquency rate for multifamily loans held by insurance companies is only 0.15 percent (MBA, 2012). While the study of mortgages has always been

important to the financial industry, the “Great Recession” highlighted the need for all participants in the market to understand the detailed characteristics of a mortgage and how those characteristics will change the way the mortgage responds to economic and financial conditions.

Any mortgage is a promise to repay a loan backed by the property and potentially other assets through a recourse agreement. Therefore, it is not a surprise that the study of commercial and multifamily mortgages has utilized many of the concepts and lessons learned in the extensive body of single family mortgage literature.

Mortgages can terminate through two different avenues – 1) the mortgage is paid off on time or early, or 2) the borrower stops making payments and the mortgage goes into default. In the commercial mortgage market the primary motivation to pay off a loan early is to replace existing debt with cheaper debt or to sell the property to a new owner. The interest rate motivation is mainly driven by changing market interest rates or perhaps changes (only improvements) in the risk of the property that provide access to lower cost debt. There is very consistent evidence in both the residential and commercial mortgage markets that falling interest rates can drive early terminations of mortgages as borrowers refinance their debt (For example, Ciochetti, Deng, Gao, and Yao 2002 and Abraham and Theobald 1997). Prepayment penalties are often used in the commercial market to discourage prepayment and preserve the return to the lender, and there is strong evidence that such penalties are very effective in suppressing prepayments. Once the penalty expires, of course, prepayments spike up dramatically. For example, Fu, LaCour-Little, and Vandell (2003) show that conditional prepayment rates can jump from almost 0 percent right before the penalty expires to 50 percent in the following month. This has been coined the “hockey stick” shape of prepayments. Alternative mechanisms for suppressing prepayments include lock outs (no prepayment allowed) and yield maintenance requirements. The typical yield maintenance agreement requires the borrower to replace the expected mortgage payments with like-kind US Treasury expected payments. Key issues the literature does not address are whether and how prepayment suppression mechanisms affect default probabilities, and whether commonly

accepted financial incentives to prepay and default still work when prepayment penalties and yield maintenance agreements are in effect.

The second source of early termination is default, when mortgage payments are no longer being made. One motivation for default exists when the mortgage is worth more than the property. This is often called negative equity. Most research proxies for negative equity by examining property prices, the Loan to Value (LTV) ratio at origination, or updated current value using market statistics (the current LTV or cLTV). Ignoring transaction costs, the single family borrower is called “ruthless” if the default option is exercised whenever the cLTV is greater than 100 percent (Foster and Van Order 1984 and 1985). “Trigger events” such as the death of a family member, divorce, illness, and unemployment, can also motivate a borrower to default by making it hard, financially, for the borrower to service the debt.

The empirical evidence for commercial property loans suggests that the equity position of the property in the contemporaneous time period, not the equity position at origination, is a good proxy for the default option (Ambrose and Sander 2003, Archer et al 2002, and Cichetti, Deng, Gao, and Yao 2002). Most research has considered the ability of the property’s income/cash flow to cover debt service obligations as the best analogy to home mortgage “trigger events.” For income-producing property (as contrasted with single family property) the property is the main collateral (assuming the loan is non-recourse) and is the source of income to cover the debt. The Debt Coverage Ratio (DCR) is typically defined as the Net Operating Income (NOI) of the property divided by the Debt Service ( $DCR = NOI/DS$ ). If the DCR is less than 1.0 then there will not be enough income to cover the mortgage payments. Empirical commercial mortgage papers mostly find that higher DCR reduces the probability default (Archer, Elmer, Harrison and Ling 2002 , and Cichetti, Deng, Gao, and Yao 2002, Cichetti, Deng, Lee, and Shilling and Yao 2002, Goldberg and Capone 2002, Vandell, Barnes, Hartzell, Kraft and Wendt 1993).

The effect of equity (LTV) and debt coverage (DCR) on default is likely to be highly non-linear. Some research has used quadratic terms to highlight the lack of linearity (Cicchetti, Deng, Gao, and Yao 2002, Cichetti, Deng, Lee, and Shilling and Yao 2002) and others have focused on the interdependence of equity and debt coverage (Goldberg and Capone 2002). For



example, the option to default may not be exercised even when there is negative equity because the cash flow from the property may have substantial positive net value. Therefore, it may be necessary to have both a negative cash flow ( $DCR < 1.0$ ) and negative equity ( $LTV > 100$  percent) to make the default option in the money.

While single family loans that did not fully amortize became more popular during the subprime era, and some research ensued (Quercia, Stegman, and Davis 2007 and Renault 2004), balloon payment loans are and were much more prevalent in the commercial market. In fact, almost all the loans used in this study have a balloon payment. The literature has been concerned with the ability of the borrower to find new financing to cover the large balloon payment due at the end of the loan. The risks associated with balloon payments include refunding risk or extension risk (the loan lasting past the balloon date), term default (default on the balloon due date), and workout strategies on defaulted loans (minimizing losses after the default) (Chen and Deng 2012, MacDonald and Holloway 1996, Tu and Eppli 2003, and Eppli and Tu 2005). These papers emphasize, through simulation, that even if the equity position and the cash flow position are acceptable, constraints on credit availability can restrict refinancing. This becomes even more challenging when lending standards are tightened and may require a substantial infusion of capital if property prices have declined. However, the literature does not provide much evidence on default and repayment probabilities at term with a balloon payment.

This research contributes to the literature in a number of ways. First, it has been almost 10 years, as far as we can find, since an empirical study of commercial property mortgage termination has been published in the academic literature. In light of the mortgage market upheaval in the last 10 years it makes sense to revisit this arena. Second, there are only two prior research papers focused on multifamily mortgage terminations (Archer et al 2002 and Golberg and Capone 2002), and Archer et al could not find statistical significance for some key drivers of default. Moreover, prior research did not include property specific contemporaneous information to proxy for the default and prepayment options of termination. Third, little is known about how mortgage contract provisions designed to suppress prepayments affect default probabilities. This may be non-trivial because the removal of one option (prepayment) may make a second option (default) to terminate a loan more likely. It is also likely that when

the option to prepay a loan is largely suppressed through contract terms, the typical financial motives (equity, cash flows and interest rates) to prepay or default may have substantially different marginal effects on how a mortgage terminates.

### **The World of the Small Multifamily Loan**

Since its original charter in 1938, Fannie Mae has played a critical role in the U.S. rental housing market, supporting the construction and financing of economically sound rental housing projects.<sup>2</sup> Based on Fannie Mae's \$2.2 billion of small loan production and the MBA data, Fannie Mae's estimated market share for small loans in 2009 was 15%. As of midyear 2010, Fannie Mae held a \$34 billion book of 30,000 loans on properties with loans of \$3 million or less or up to \$5 million in high cost MSAs (18 percent of total multifamily book) or a \$21 billion book of 23,500 loans on five- to 50- unit properties (12 percent of multifamily book). Roughly 86 percent of Fannie Mae's 2009 small loan book of business was affordable to families at or below 100% area median income and met the definition of affordable housing set forth by the U.S. Department of Housing and Urban Development (HUD) (FNMA, 2011). Fannie Mae's segment of small loans performs very well with most cohorts by year of origination, defaulting at cumulative rates under 0.15 percent. The cumulative paid off rate for loans is highest from cohorts 10 years or older and at or about 6 to 7 percent.

In general, the market defines small multifamily mortgages by property and unit count and loan amount. A small loan is defined as a mortgage loan for an apartment building with five to 50 units or a loan with a principal balance of \$3 million or less in most markets, or up to \$5 million in high cost metropolitan areas. Fannie Mae uses the principal balance definition of \$3 million or less nationwide and \$5 million or less in high cost markets like New York City and Los Angeles. A small loan is not always synonymous with a small property. Limiting the definition of small loans to properties with five to 50 units results in the exclusion of larger subsidized affordable multifamily properties. These larger, subsidized properties also generally benefit

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<sup>2</sup> Fannie Mae became involved in the multifamily market as part of the New Deal when the federal government created its own mortgage association to facilitate construction and financing of rental and for-sale housing, by making direct loans insured by the Federal Housing Administration. Over time Fannie Mae's authority expanded and in 1984 Fannie Mae created a distinct business division to purchase multifamily loans (Fannie Mae, 2012).

from the low income housing tax credit (LIHTC), which offers subsidies that reduce the owner's debt load when the owner offers some below market rents to qualified tenants (FNMA, 2011).

Fannie Mae describes the typical small loan borrower and mortgage according to the following:

- Small loan borrowers often self-manage properties. In September 2010, 57 percent of Fannie Mae small loan borrowers employed professional property management compared to 85 percent for non-small loans.
- The small loan borrower's ability to repay is driven by the strength of the property cash flow, as well as the borrower's own financial strength and repayment history.
- According to the Mortgage Bankers Association there are more than 2,600 lenders, each originating a small number of loans. This makes standardization and efficient underwriting more challenging.

### **Empirical Approach**

As early as 1981 when Dunn and McConnell presented their mortgage backed security model, and in 1986 when Green and Shoven's model examined the impact of market conditions and loan prepayment, researchers have recognized that mortgage contracts are best modeled in a contingent claims framework. Each period the borrower faces a decision with three options -- prepay, default or continue to make payments. Motivations for each option distinguish one from the other. For example, the borrower's option to prepay the mortgage at any time without penalty is a call option at a strike price of par while the default option is a put option at a strike price equal to the market value of the collateral property (Ambrose and LaCour-Little, 2001). The multifamily mortgage is further complicated by the inclusion of prepayment penalties that, as will be illustrated, do influence the borrower's decision to default.

The model used to estimate default incidence is based on an optimization model of consumer choice similar to that established by Campbell and Dietrich, 1983 and since used extensively in residential mortgage research (see Pennington-Cross, 2010 for example). During each period in the life of the mortgage the borrower chooses the status of the mortgage. The borrower is assumed to maximize a utility function defined over a vector of mutually exclusive qualitative choices,  $S$ , which in our case is prepay, default or continue to make payments, and a vector of exogenous state variables,  $X$ . It is assumed that the utility maximizing choice can be represented as a probability function of the state variables,

$$P(s_j | X) = \frac{e^{\alpha_j + \beta_j X}}{\sum_{k=1}^n e^{\alpha_k + \beta_k X}} \quad (1)$$

where the sum of the probabilities of all  $n$  elements in  $S$  for a given  $X$  is equal to unity such that:

$$\sum_{j=1}^n P(s_j | X) = 1 \quad (2)$$

and  $s_j$  represents the  $i^{\text{th}}$  choice variable of the representative borrower. The qualitative choice problem is to specify the set of choices,  $S$ , and the exogenous state variables,  $X$ , which determine the utility maximizing choice, and to restrict the form of the function,  $f_i(X)$ .

Given the three states (default, prepayment, or active) in which a mortgage may be observed in a given period let  $Y_{im} = 1$  if the  $i$ th observation chooses alternative  $m$ ,  $m = 1, 2, 3$ . In the case of this analysis  $i$  represents individual mortgages and  $m$  represents one of 3 states (in no particular order at this time) (Pinder, 1996). Let  $\pi_{im} = P(Y_{im} = 1)$ ; then for a 3 state model we have:

$$\pi_{im} = \frac{e^{\alpha_m + \beta_m X_i}}{\sum_{k=1}^3 e^{\alpha_k + \beta_k X_i}} \quad (3)$$

or more generally as:

$$\pi_{im} = \frac{e^{\alpha_m + \beta_m X_i}}{\sum_{k=1}^m e^{\alpha_k + \beta_k X_i}} \quad (4)$$

for the general case with  $m$  alternative states for the dependent variable. Incorporating this dependent variable into a multinomial logit specification the model results in the following:

$$\ln \left( \frac{\pi_{im}}{\pi_{i1}} \right) = \alpha_m - \alpha_1 + \beta_m X_i - \beta_1 X_i \quad (5)$$

In a three-state model, solving for  $(\alpha_2 + \beta_2 X_i)$  and  $(\alpha_3 + \beta_3 X_i)$  provides the following:

$$\ln \left( \frac{\pi_{i2}}{\pi_{i1}} \right) = \alpha_2 - \alpha_1 + \beta_2 X_i - \beta_1 X_i \quad (6)$$

$$\ln \left( \frac{\pi_{i3}}{\pi_{i1}} \right) = \alpha_3 - \alpha_1 + \beta_3 X_i - \beta_1 X_i \quad (7)$$

$$\ln \left( \frac{\pi_{i2}}{\pi_{i3}} \right) = \alpha_2 - \alpha_3 + \beta_2 X_i - \beta_3 X_i \quad (8)$$

$$\ln \left( \frac{\pi_{i2}}{\pi_{i1}} \right) = \alpha_2 - \alpha_1 + \beta_2 X_i - \beta_1 X_i \quad (9)$$

$$\ln \left( \frac{\pi_{i3}}{\pi_{i1}} \right) = \alpha_3 - \alpha_1 + \beta_3 X_i - \beta_1 X_i \quad (10)$$

where state one is the benchmark for comparison (Long, 1997), in the case of this analysis state 1 is active, and  $\alpha_1 = \beta_1 = 0$ .

Tables 1-3 list the state variables that will be tested. They are broadly grouped into: 1) Basic, which includes standard information about the loan that is known at origination and does not vary over time, 2) Ratios, which includes ratios indicating the equity and cash flow positions of the property, 3) Market, which includes proxies for market conditions in the local labor and property markets, and national interest rates, 4) Prepayment, which includes a series of variables designed to capture the temporal impact of prepayment penalties, yield maintenance provisions, and balloon requirements on default and prepayment/term repayment, 5) Building and Operations, which describes the characteristics of the building and its operations, and lastly, 6) Product Channel, which describes where the loans come from and the type of underwriting the loans underwent.

## **Data**

Each observation in our sample describes a single multifamily mortgage securitized or held in portfolio by Fannie Mae and originated between 2005 and 2011. Fannie Mae purchases individual loans originated by approved mortgage lenders or purchases pools of already securitized loans. Servicing remains with the lender and, like underwriting, is conducted according to guidelines prepared by Fannie Mae. Lenders retain a risk position in these loans through a loss sharing agreement with Fannie Mae. Presently, Fannie Mae multifamily purchases must allow for the securitization of the loans into Fannie Mae guaranteed mortgage-backed securities (MBS) and the sale of those MBS to investors. Typically, Fannie Mae restricts purchases of small loans to standardized mortgages with a 10 year term, fixed rate and prepayment penalty.

This is a panel dataset where quarterly observations are followed through the fourth quarter of 2011. Our goal is to find the determinants of the conditional prepayment probability and default probability for these mortgages. The term conditional indicates that we are studying the probability in a current quarter conditional on it surviving all the prior quarters.

Figure 1 illustrates the cumulative default rate (percentage of loans that default) for different cohorts of loans grouped by origination year. There are few loans that have entered the default state. Of those loans that have defaulted, cohorts 2000, 2004 and 2006 stand out as higher than normal years. There appears to be censoring in the data given there are no defaults

observed in 2010 and 2011. This is likely due to the fact that the movement to default in the multifamily mortgage market generally occurs over the course of years, rather than taking mere months as was the case for subprime residential loans. A second and equally plausible explanation for the lower default rate is that both Fannie Mae, and the financial markets at large, significantly elevated underwriting standards and thresholds beginning around 2009. The censoring challenge is clearly observable in Figure 2 where we plot the cumulative percentage of loans by cohort and yield maintenance provisions. Censoring aside, the proportion of observed loans that end in prepayment is relatively small; for instance, only 6.3% ending in prepayment for loans originated in 2000. There is an important element in many of the observed loans that explains some portion of this phenomenon. Almost all loans are balloon loans with a term averaging just over 10 years (123 months). In essence between the prepayment penalty, the yield maintenance provision, and the balloon payment/refinancing requirement, there is only a relatively small period of time when prepayment is a viable alternative.

A data dictionary is presented in Tables 1-3. These tables include all the variables utilized in the various specification tests, providing the variable name and a description of the variable. If the source for the data is anything except the Fannie Mae serving platform, the source is also noted. As previously noted the panel is restricted to “small loans” as defined by Fannie Mae. After cleaning and coding, the working dataset contains roughly 10,100 loans and 166,900 observations. The variables are organized into six different groups.

The first group, “Basic,” includes the loan age in months (*loanage*) and loan age squared (*loanage2*). This will be used to estimate a common baseline conditional quarterly probability of default or prepayment.<sup>3</sup> The size of the loan is included (*upb\_orig*) to proxy for fixed costs or the sophistication of the owner/borrower. An indicator of whether the loan is an origination or a refinance (*refi*) is also included. Table 4 indicates that most of the loans are refinances. Since there is a longer loan history to review when underwriting a refinance, it is likely that these loans may default less frequently. The majority of loans, over 90 percent, have a recourse

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<sup>3</sup> Loans are coded as defaulted if they are liquidated due to foreclosure, deed in lieu of foreclosure, repurchase, dissolution, or a discounted payoff. Loans are coded as paid off or prepaid if the loan is fully paid due to refinancing, repayment at term, or a third party sale. This grouping is necessary due to the low frequency of terminations for many of these categories.

provision with the lender or the owner (*recourse*). Recourse rights may make the borrower more conservative and dampen termination probabilities. The risk spread of the mortgage rate over the 10-year constant maturity Treasury rate (*t\_spread*) has been found in some prior research to be a proxy for unobserved characteristics. If it does function in this manner, a higher risk spread should increase default probabilities and decrease prepayment probabilities. Lastly there are indicators for whether the loan is interest only (*io*) or adjustable rate (*arm*). Very few loans have adjusting rates and approximately 10 percent have interest only features.

The second group, “Ratios,” includes various measures of the property’s equity and cash flow positions. The two measures of the DCR are at origination (*dcr\_acq*) and the current or updated DCR using NOI (*dcr\_noi*). The prior literature usually finds that higher DCRs tend to reduce defaults and increase prepayments. Unlike most prior research, which has had to estimate updated values, we observe the actual NOI for the property over time (annually); thus, we do not need to use any market indexes to update the debt payments and income. The last two ratios report the LTV at origination (*ltv\_orig*) and the current or updated LTV (*cltv*). Prior research has found that LTV at origination provides little information, but a higher current LTV should drive default probabilities up and prepayments down. On average these small loans have a lot of equity with *ltv\_orig* equal to 0.586.

Table 2 continues the data dictionary for the third group of variables called “Market.” These variables include measures of market conditions. In general, if market conditions are better we should expect the property to perform better and generate more income. Measures of the local unemployment rate (*urate*), changes in multifamily rents over the prior quarter (*Δrent*), quarterly changes in capitalization rates (*Δcap*), and changes in multifamily interest rates (*Δirate*) are included. The prior literature has often relied on market conditions to proxy for unobserved property conditions (vacancy, income, efficiency). However, in this paper we include both market and observed property specific conditions. In addition, we do expect that rising interest rates should suppress prepayments, but this impact may be muted by mortgage contract features.

The fourth group, “Prepayment,” includes various measures of contract provisions that could substantially suppress or encourage prepayments. Since the timing of the provisions can

be very important a series of dummy variables were created for each provision. For example, the yield maintenance provision is described using four different dummy or indicator variables. The variable *ym* indicates all time periods until the provision expires. Then a series of dummy variables were tested to look for patterns in prepayment and default around the date that the yield maintenance provision expires. *ym\_t01* is a variable indicating the quarter in which the provision expires in ( $t=0$ ) and the quarter before the provision expires ( $t=1$ ). In addition we test for differential impacts around the expiration of the yield maintenance provision in two quarters ( $t=-1$  and  $t=-2$ ) after the provision expires, *ym\_tn2*. This is a time when prepayments likely increase rapidly. There may also be some lasting impacts so we test for the impact of time periods that are three or more quarters after the expiration of the provision ( $t\leq-3$ ), *ym\_tn3*. Various specifications were tested but these time periods had a sufficient quantity of termination events to be empirically estimated.

This same approach is used to address prepayment penalty provisions. The variable *paypen* indicates all time periods until the prepayment penalty expires. We include dummy variables indicating that the provision expires in the current quarter or one quarter into the future ( $t=1$  and  $t=0$ ), *paypen\_t01*, or that the provision expired in the prior quarter ( $t=-1$ ), *paypen\_tn*. Again, this is designed to look for differential prepayment behavior as the penalty is about to expire and after it does expire.

A dummy variable is used to indicate which loans have a balloon feature, *balloon*. Unlike *paypen* and *ym* this variable does not have a temporal component. To capture differential impacts as the balloon payment comes due we include a dummy variable indicating that the balloon payment is due in the current quarter, *balloon\_t0*, and another dummy variable indicating that the balloon payment is due in one more quarter, *balloon\_t1*.

In Table 3, the fifth group, "Building and Operations," examines characteristics about the building itself and how it is performing. The size of the property, proxied by the number of units, *b\_units*, may impact the owner's ability to survive economic downturns or to access credit markets. An older property, proxied by the year the building was built, *year\_built*, may also become economically obsolete as market household preferences change or location characteristics change. A proxy for the overall vacancy of the property is created by dividing the



annual effective gross income (*egi*) by the potential gross income (*pgi*), *egi/pgi* for the property. The effective gross income is the total income of the property including rent and non-rent income. The potential gross income proxy is the total income of the property should receive if all occupied units are rented at their current actual rents and all vacant units are rented at current market rents. We expect that property that is fully rented should have an easier time meeting its financial obligations and accessing the credit markets. A measure of operating efficiency is calculated by dividing operating income by effective gross income, *oper%*. A measure of capital expenditure is calculated by dividing capital expenditures by effective gross income, *capex%*. A measure of reserves is included as the dollar amount of reserves at the beginning of the year, *reserves*. The expected impact of these measures of property operations is an empirical question.

In Table 3 the sixth and final group, "Product and Channel," indicates where each loan comes from and the type of underwriting utilized. Small multifamily loans largely are underwritten through prior approval, *pa*, the delegated underwriting and servicing platform, *dus*, and the multifamily flex program, *mflex*. Under these platforms the loans can be acquired as a cash loan or they may already be in a mortgage backed security, *mbs*.

Table 4 presents summary statistics for the variables and a number of interesting observations. For example, the mean debt coverage ratio at origination for the observations is 2.32, which is very high and may reflect tighter underwriting of smaller loans. The mean LTV at origination is 59 percent and as expected the current LTV (over the observed time period) falls to 55 percent. Following industry standards the yield maintenance and prepayment penalties affect almost all loans at some point in their lives and almost all loans have a balloon feature. 10 percent of the loans are interest only while 92 percent have a recourse component built into the mortgage contract. The number of units ranges from 1 to 148.

Figure 3 provides an illustration of loan terms (left axis) and the original interest rate (right axis) over time. The mortgage rate follows a path similar to the residential mortgage market over the same period. After a precipitous decline over the period 2000 through 2003, there is an increase consistent with the quarterly increase in the federal funds rate during 2005 and 2006. From 2006 through 2011 average rates hover between 5 and 6 percent. There is a

great deal of variation in the term of the observed loans with an average high of over 150 months in 1998 and an average of 108 in 2008.

In addition to static observations at origination we have variables that are dynamic over the observed tenure of the loan. The loan age, or vintage, runs from 1 to roughly 201 months. For the financial variables, the expense percent indicates there are mortgages with expenses exceeding NOI by as much as 172 percent. The variable *capex%* suggests that actual capital expenditures are on average very low compared to effective gross income but there are extreme cases where capital expenditures are multiples of effective gross income. The mean for the Treasury spread, *t\_spread*, is 260 basis points with a range from negative to 412. Figure 4 is a dual scale graph with the loan to value ratio (LTV) at origination on the left axis in columns and the spread between the original contract interest rate and the Treasury rate on the right axis. Although the time span is relatively short there is an observable inverse relationship between the spread and the average LTV of the loans, 2007 through 2011. Higher spreads are present in years when the average LTV is lowest. Likewise, in the late 1990s and early 2000s, when LTVs are averaging over 65% the spread is between 100 and 200 basis points. The changing perceptions of risk in real estate investments and the appetite for real estate in the securities market can explain part of this. Another explanation is the extremely low Treasury rates during the latter years.

We also control for economic conditions. The state unemployment rate ranges from 3.17 to 13.63 over the observation period. This is state level data due to the challenge in obtaining county level estimates over the panel period. We use the Housing and Urban Developments Fair Market Rent estimates by county to obtain an estimate of the change in median rent for a 2 bedroom apartment each year. As the table indicates the mean annual change is 2.8 percent.

In Figure 5 we consider delinquency. The mean days delinquent, contingent on being delinquent, is illustrated by the bars and scaled on the left axis and percent of outstanding loans that were delinquent is the line on the right axis. For loans that are delinquent the average number of days exceeds 170 beginning in the first quarter of 2011. It is interesting that this rapid increase did not occur until 2011. A continuation of the data would likely result in an

increase in foreclosures in 2012 and forward as mortgages delinquent in excess of only 90 days are considered seriously delinquent (i.e. approaching default).

## **Results and Implications**

Table 5 shows the base specification results for the quarterly multinomial logit using panel data with three outcomes – continue, default, or pay off the loan. Consider first the results for default. There is an increased probability of default as the vintage of the loan increases but, as indicated by the *loanage2* variable, at a decreasing rate as time passes. This is illustrated in Figure 6 and shows the peak of the baseline at 102 to 108 months of the loan or the 8<sup>th</sup> to 9<sup>th</sup> year of the loan's life. Longer term loans tend to default less in each quarter and interest only loans tend to default at a much higher clip. The term effect is illustrated in Figure 7 and highlights that loans with long terms have very low default rates. Table 6 lists the expected probability of default for the dummy variables when they are 0 and when they are 1. For example, an interest only loan on average has a 0.19 percent probability of defaulting in a quarter while a loan with amortization features has a 0.10 percent chance of defaulting in any quarter. The effect of loan features such as having recourse versus non-recourse and purchase versus refinance is very small. Oddly enough, Figure 8 shows that having more units in a building increases default likelihood. The inverse would be intuitively expected since having fewer units should tend to increase the volatility in cash flows.

Archer et al (2002) found that LTV at origination has no relationship with default while DCR at origination does. By contrast, as described in more detail later in this paper, our results show a significant effect for LTV at origination but not for DCR at origination.

For the prepayment option, loan age is again positive and significant. As shown in Figure 6 the probability of the loan being paid off increases at an increasing rate even after controlling for the term of the loan, prepayment suppression provisions, and balloon timing. Compared to fully amortizing loans, loans with a balloon feature are much more likely to be prepaid, see Table 6, in any time period. Table 6 also illustrates the effectiveness of yield maintenance and prepayment penalty provisions in reducing the rate of prepayments. For example, on average, after a yield maintenance provision expires the probability of the loan being paid off jumps from 0.74 percent to 5.25 percent. A similar shift is also found when the prepayment penalty

expires (we will discuss this further in later sections). The impact of product and channel is not very consistent across the specification tests so we will provide little discussion of those results.

### *Prepayment Provisions*

In Table 7, we focus on the expiration of yield maintenance and prepayment provisions and the timing of the balloon payment. First consider loans with balloon payments. Loans with balloon payments are less likely to default for most of the loan's life. However, as the balloon payment approaches, default probabilities increase very quickly. This is shown through the coefficient estimates for the various balloon dummy variables and is easy to see in Figure 10. This figure plots the average probability of default for 10-year term loans as they approach the balloon payment in month 120 and compares it to another 10-year loan without the balloon payment. The loan payments and all other characteristics are the same for both illustrations. For the loan without the balloon payment the quarterly conditional probability of default is almost flat. For the balloon loan there is a strong spike from less than 0.5 percent to almost 5 percent as the balloon payment arrives. There is a similar path for loan prepayment or paying off the loan at term. As shown in Figure 11, the probability of the loan being paid off increases from about 4 percent to 15 percent one quarter before the loan is due and reaches a peak of 45 percent at loan term. So, conditional on the loan being alive when the balloon payment is due the model estimates that on average just 45 percent of the borrowers will successfully make the payment. Therefore, there is substantial extension risk associated with balloon payments.

Figures 12 and 13 provide the same type of analysis but examine what happens to the average probabilities of default and prepayment as a 3-year prepayment penalty expires. There is a spike in defaults in the quarter before the penalty expires and at expiration. Defaults are also elevated, relative to a loan without prepayment penalties, for all prior time periods. The spike in prepayments occurs directly after the penalty expires with a doubling of the probability.

Figures 14 and 15 repeat the analysis for the 9.5-year yield maintenance provision expiration (month 114). In terms of default, the pattern is similar. Relative to a loan without a yield maintenance provision, the probability of default with a yield maintenance provision is always elevated (before, at, and after expiration of the provision). There is also a large spike in

default probabilities around the expiration date. The probability of prepayment or paying off the loan also increases around the expiration date.

These results indicate that such provisions can successfully suppress prepayments while they are in effect, but this benefit carries a significant cost: with lower prepayment rates default rates spike near the expiration date of the restriction.

### *Market Conditions and Performance Ratios*

Table 8 includes indicators of labor market, space market, capital market (prices), and interest rate conditions. Interest rates perform as expected: if interest rates have decreased since origination then the probability of the loan prepaying increases. The local unemployment rate also functions as expected: when labor market conditions are worse (higher unemployment rates) the probability of default increases and the probability of prepayment decreases. However, changes in market level pricing of property and rental rates are statistically insignificant and inconsistent across various specifications. One interpretation is that the thing that matters is the building itself and how it is performing along with the features of the loan, not how the local region is performing.

### *Endogeneity*

Table 9 adjusts the DCR and LTV ratios from what was observed at origination to contemporaneous values. This should help to disentangle the endogenous relationship of these ratios identified by Archer et al (2002). The results indicate that the contemporaneous or current LTV (*cltv*) and the contemporaneous or current DCR (*dcr\_noi*) perform well. Figure 16 illustrates the impact of these variables on average default probabilities over a large range of values. In particular, any increase in the *dcr\_noi* reduces default probabilities substantially until the ratio is very large. While the marginal impact of *cltv* is not as large, low *cltv* is associated with very few defaults and high *cltv* (negative equity) is associated with higher default probabilities. In addition, Figure 17 illustrates that prepayment is suppressed by high unemployment rates and rising interest rates.

Factors that drive prepayment probabilities up or down may have little impact if prepayment restrictions (yield maintenance requirements and prepayment penalties) are very

strong and effective. For example, if a property is hit by a negative shock making it hard to service the debt, one option is to pay off the existing loan and recast it into a more affordable loan (extend the term for example or get more equity). If prepayment is effectively suppressed, the only remaining option is to default.

Table 10 reports the average marginal effect and the standard deviation of the marginal effect across all observations. Each variable of interest is interacted in separate specifications with the yield maintenance indicator (*ym*) or the prepayment penalty indicator (*paypen*). Then the marginal impact of a one unit change in the variable is simulated as if all observations either had the provisions or did not.

The default results indicate that the magnitude of the marginal impact for both the equity (*cltv*) and cash positions (*dcr\_noi*) is larger when the yield maintenance provision exists than when it does not. Therefore, there is more sensitivity to equity and cash positions when prepayment is suppressed through yield maintenance requirements. A similar story emerges for prepayment. Before the yield maintenance provision expires interest rates (*Δirate*), equity (*cltv*) and cash (*dcr\_noi*) positions all have very small impacts on the probability of prepayment. However, after the yield maintenance provision has expired, the marginal impacts of interest rates, equity and cash positions are all substantially higher.

Unlike the results for yield maintenance provisions, the marginal impacts are mostly insignificant when examining prepayment penalties. These results are counterintuitive. Both types of restrictions would appear to create similar disincentives to prepayment, and thus we expected to see similar results.

## **Conclusion**

This paper provides the first academic research on commercial property termination in almost 10 years, as far as we can find. This paper fills that large gap by studying small multifamily loans originated from 2005 through 2011.

In general, we find that the mortgage market functions as expected and that both borrowers and lenders react to the incentives provided by local market conditions, the property's performance, and the specific contract provisions in the mortgage. The results also clarify the findings of Archer et al (2002) that LTV and DCRs at origination are endogenous and

not predictive of default in a meaningful way. We show that using contemporaneous LTV and contemporaneous DCR breaks the endogeneity. Both the contemporaneous cash flow position (DCR) and equity position (LTV) have meaningful effects on both default and prepayment of multifamily loans.

It is traditional for multifamily loans to have balloon payments at the end of the loan's life and prepayment penalties and/or yield maintenance contract provisions through different points in the loan's life. These provisions have profound impacts on the expected duration of the loans and the amount of credit risk. Both yield maintenance and prepayment penalties are largely successful in suppressing prepayments. When these provisions expire prepayments increase dramatically (often by 5 times). The main benefit of these provisions is that the expected duration of a pool of loans with similar expiration dates is fairly simple to calculate. The cost of these provisions is that they tend to increase the probability of default over the whole life of the loans, and these probabilities tend to spike up around or slightly before the expiration date of the restrictions.

Balloon payments concentrate a lot of terminations at one point of time (the balloon payment due date). Simulations show that for a 10-year term loan with a balloon payment at term payoffs rise to 45 percent of remaining loans and defaults spike to almost 5 percent of remaining loans. These probabilities imply that there is substantial extension risk associated with balloon payment loans.

In summary, the detailed contract provisions of a commercial property mortgage are extremely important. Not knowing the details of the prepayment suppression provisions and the extent of amortizing will lead to gross miscalculations of expected mortgage yields and cash flows. In addition, for these small loans it is more important to know what is going on at the property than what is happening in the market overall. The contemporaneous market value of the property, debt service, and income and cash flow generated by the property are all key determinants in avoiding defaults and understanding when prepayment will occur.

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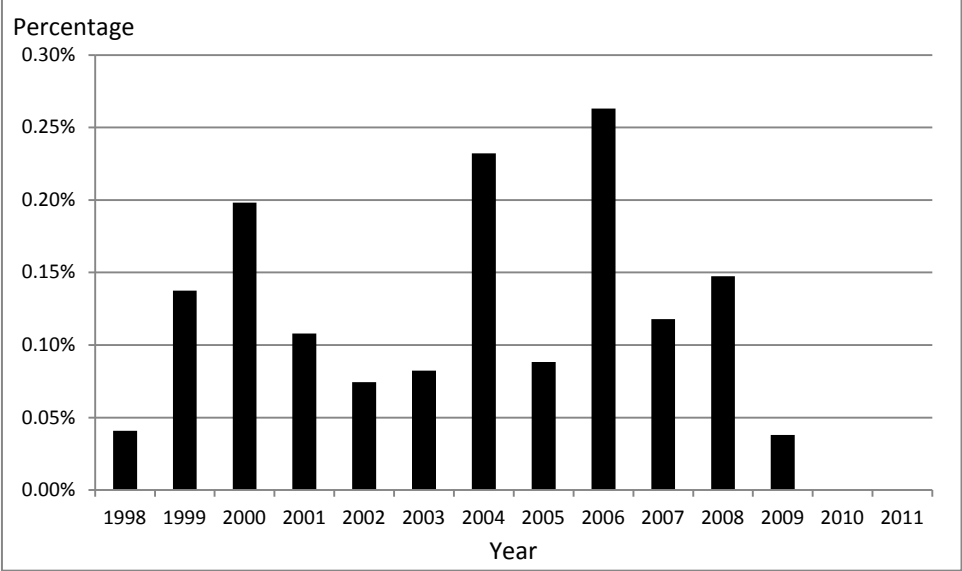
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Figures

Figure 1: Cohort or Origination Year Cumulative Default Rates



**Figure 2: Cohort or Origination Year Cumulative Prepayment Rates**

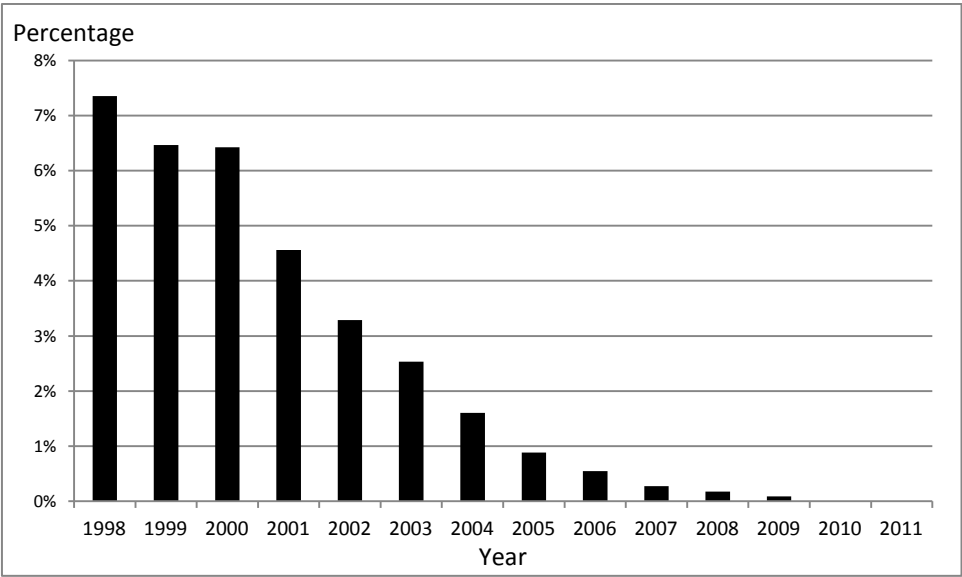


Figure 3: Evolution of Terms and Interest Rates

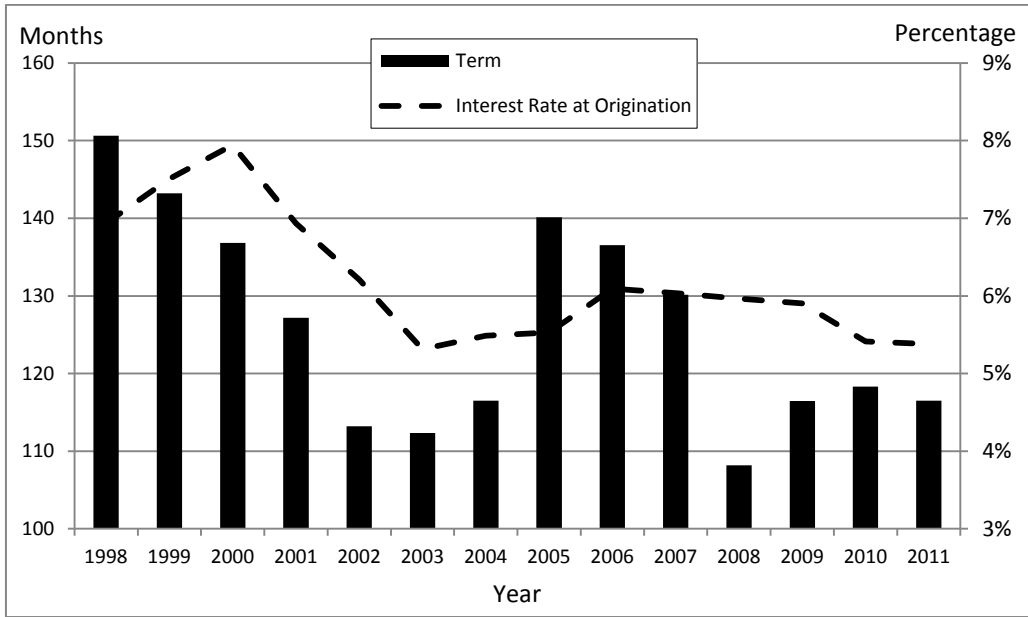
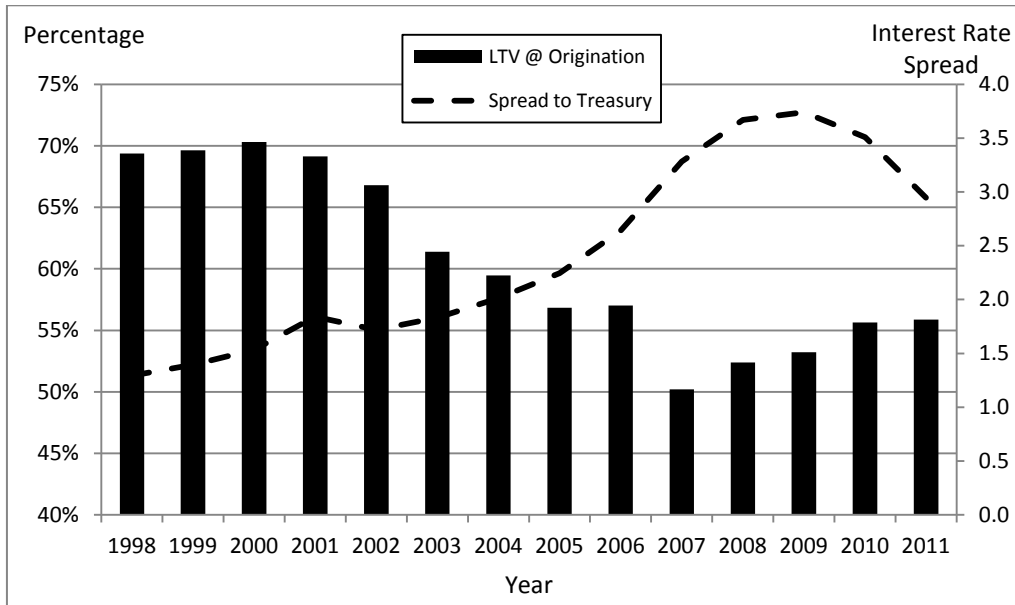
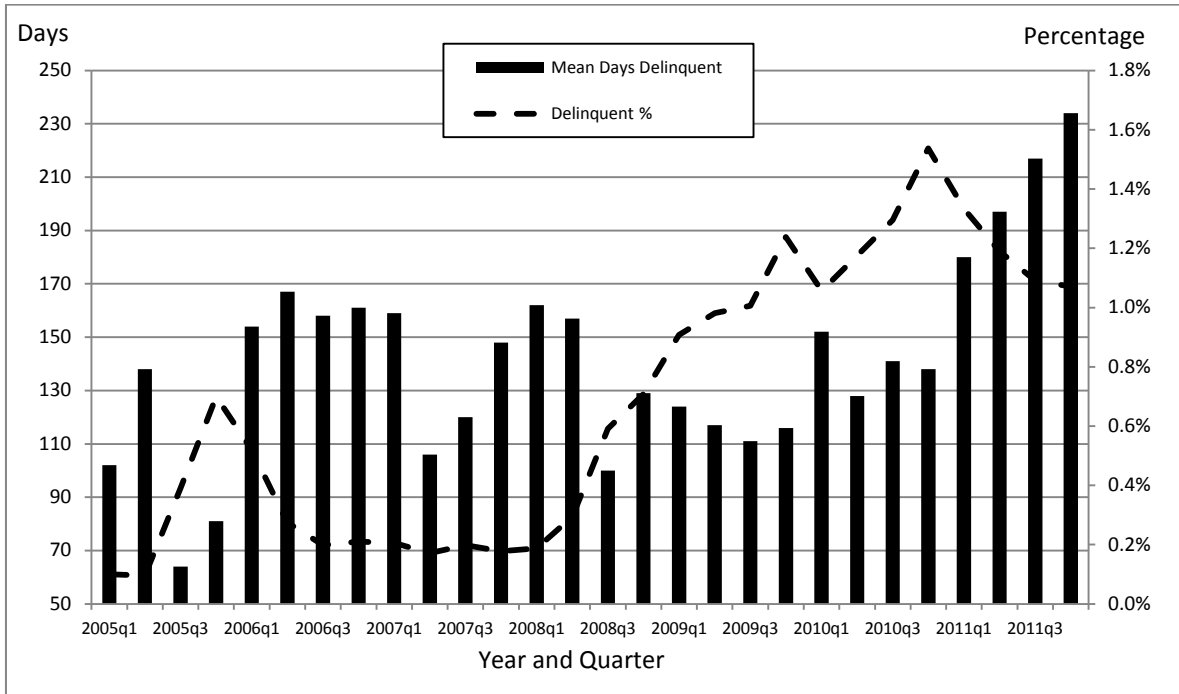


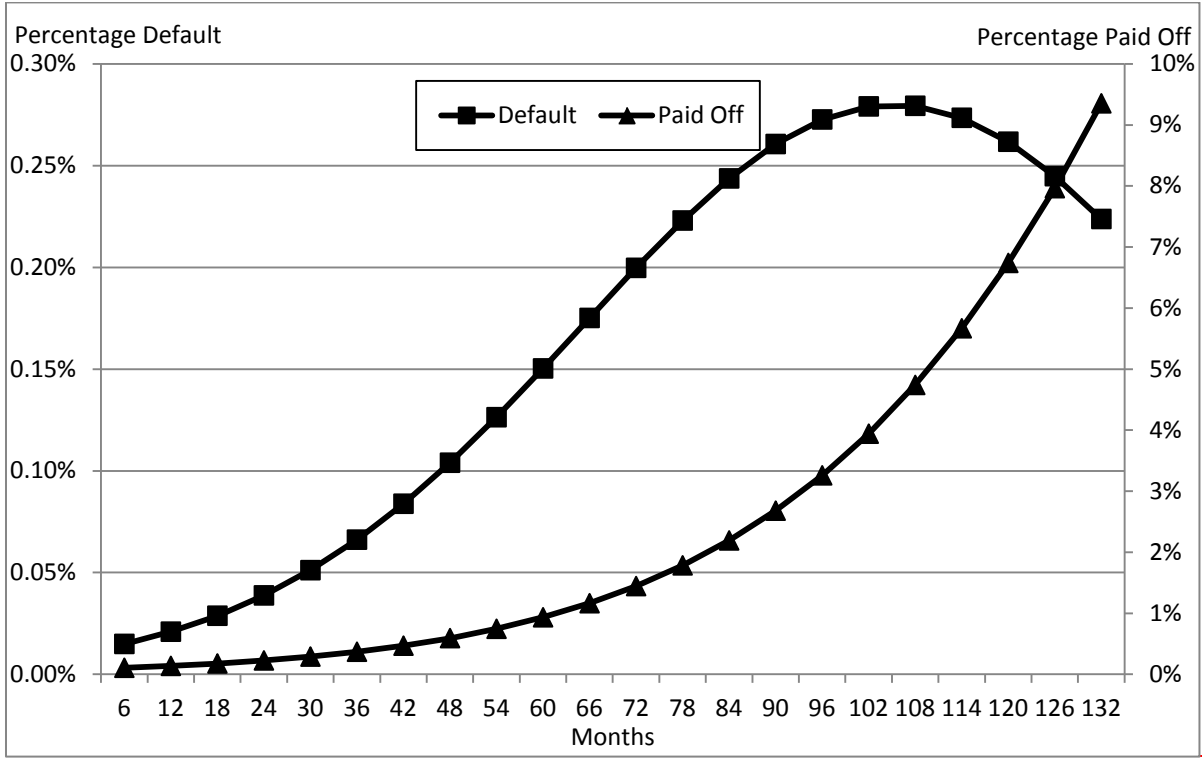
Figure 4: LTV and Spread to Treasury by Year



**Figure 5: Illustration Percentage of Outstanding Loans Delinquent and Mean Days Delinquent**



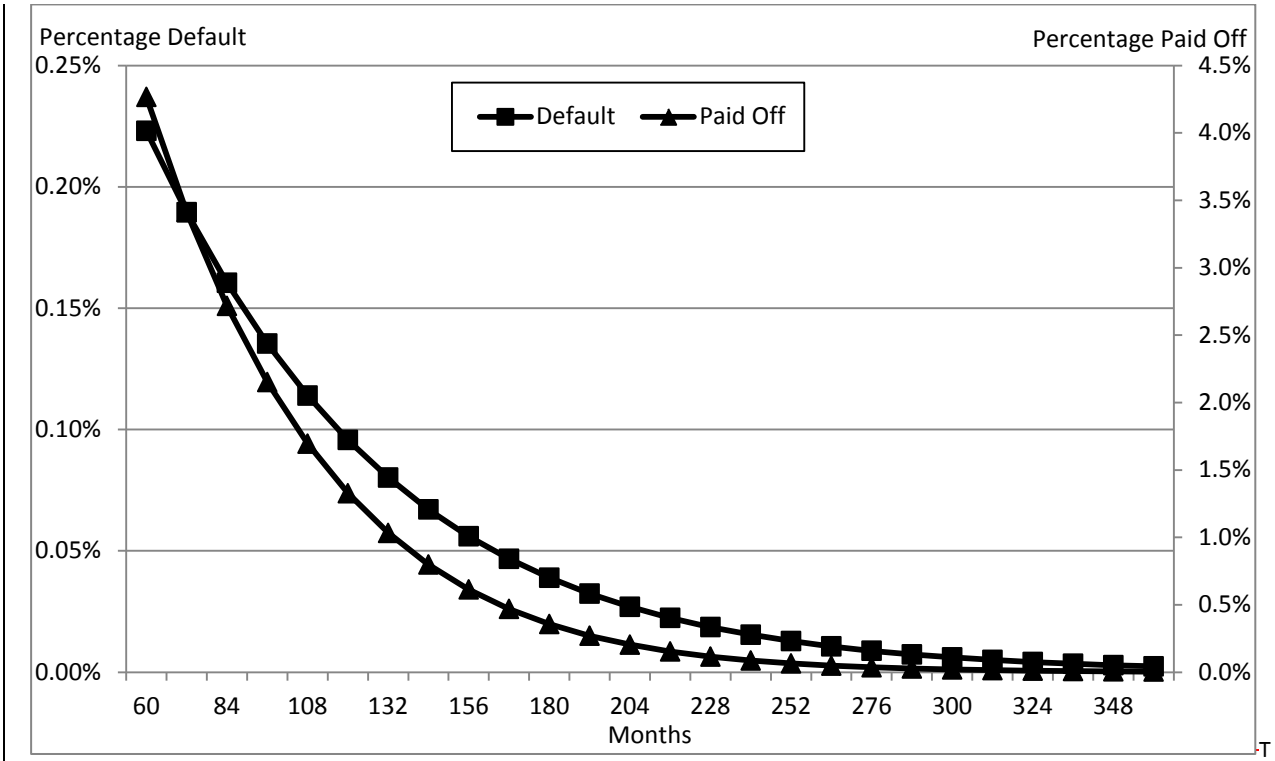
**Figure 6: Baseline Conditional Quarterly Probability**



The estimated average probability across all observations at each month indicated using the “Base Model of Early Terminations” specification.

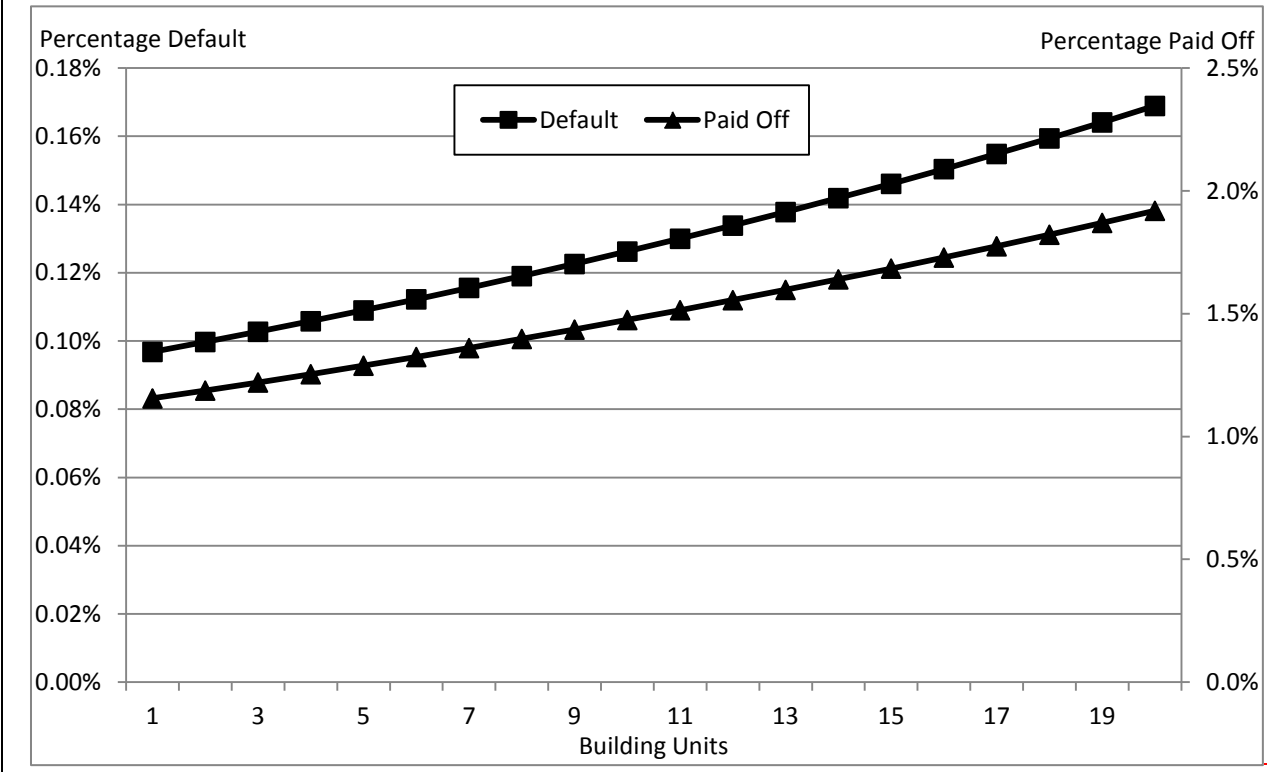


**Figure 7: Term at Origination and Termination Probabilities**



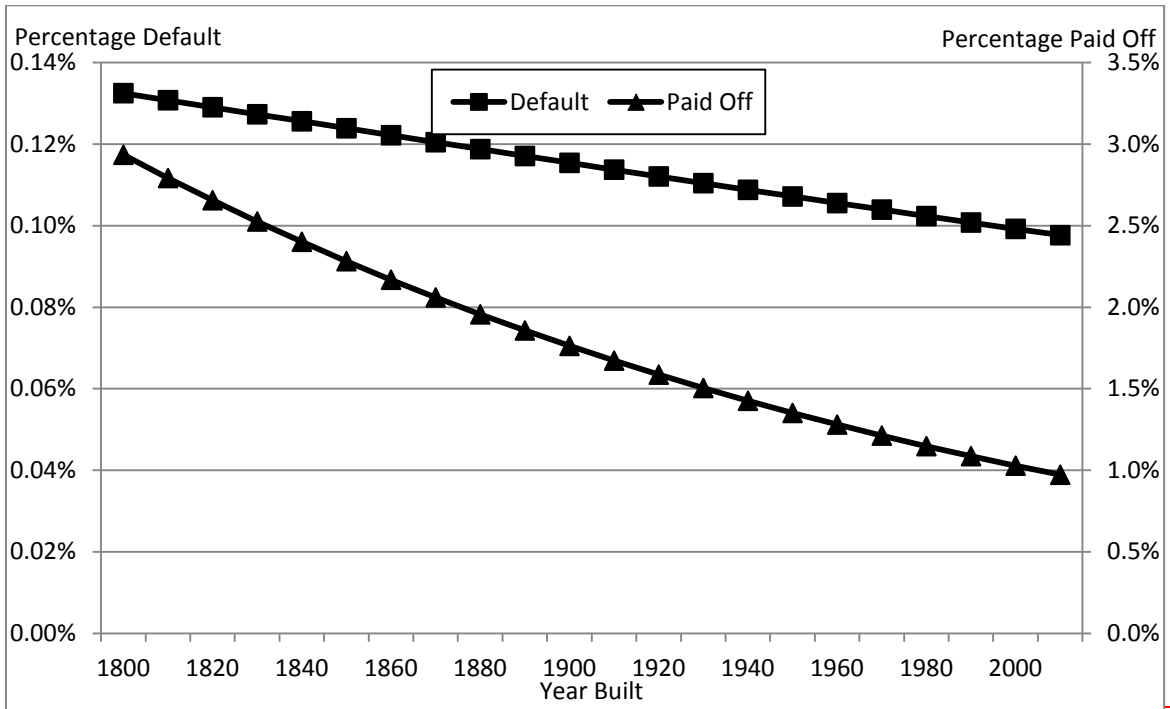
The estimated average probability across all observations at term length indicated using the “Base Model of Early Terminations” specification.

**Figure 8: Building Units and Termination Probabilities**



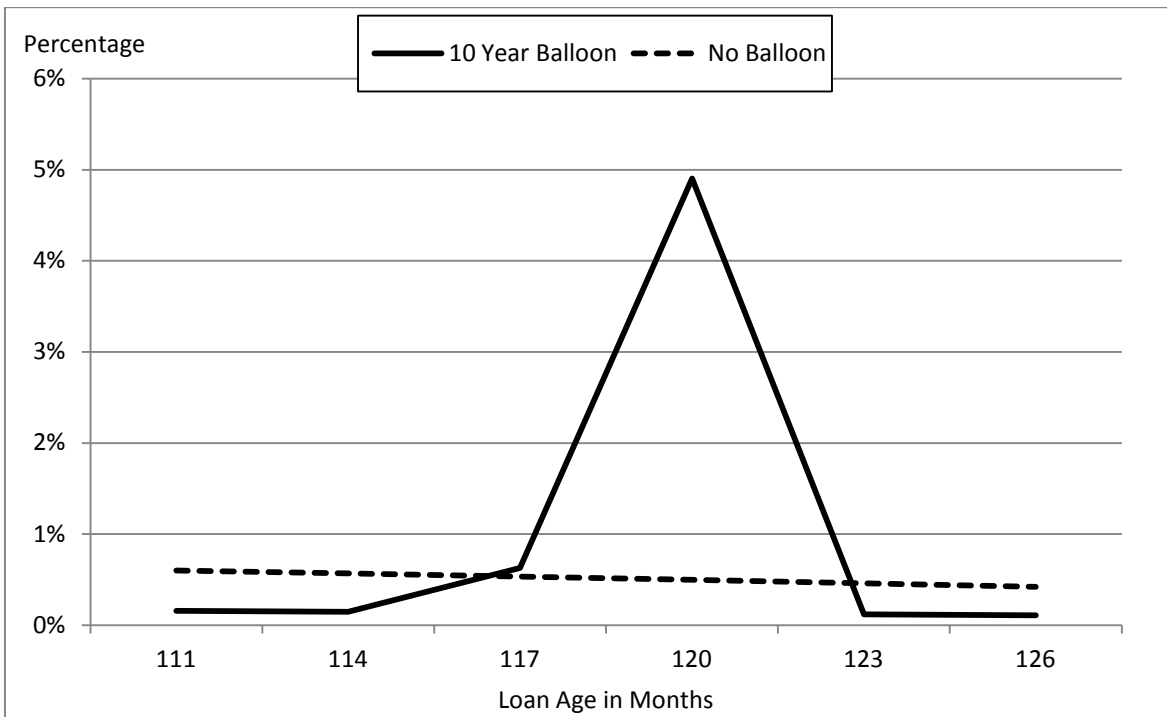
The estimated average probability across all observations at the number of building units indicated using the “Base Model of Early Terminations” specification.

**Figure 9: Year Building is Built and Termination Probabilities**



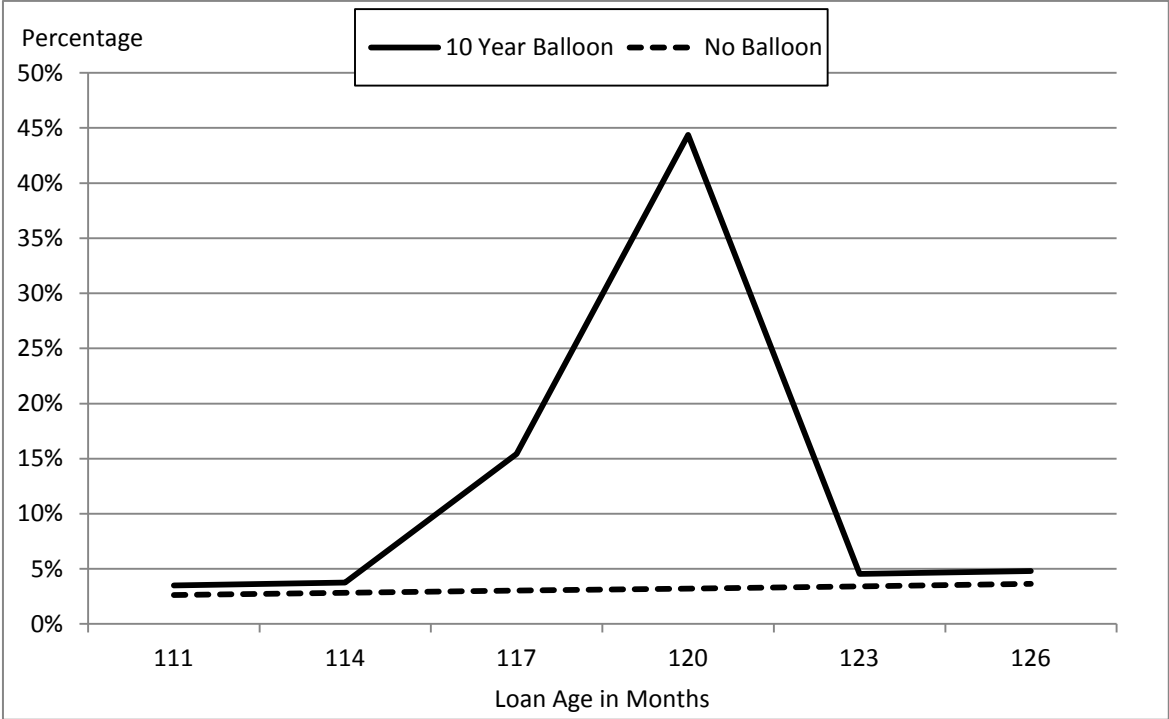
The estimated average probability across all observations at the year built indicated using the “Base Model of Early Terminations” specification.

**Figure 10: 10-Year Balloon Default Probabilities**



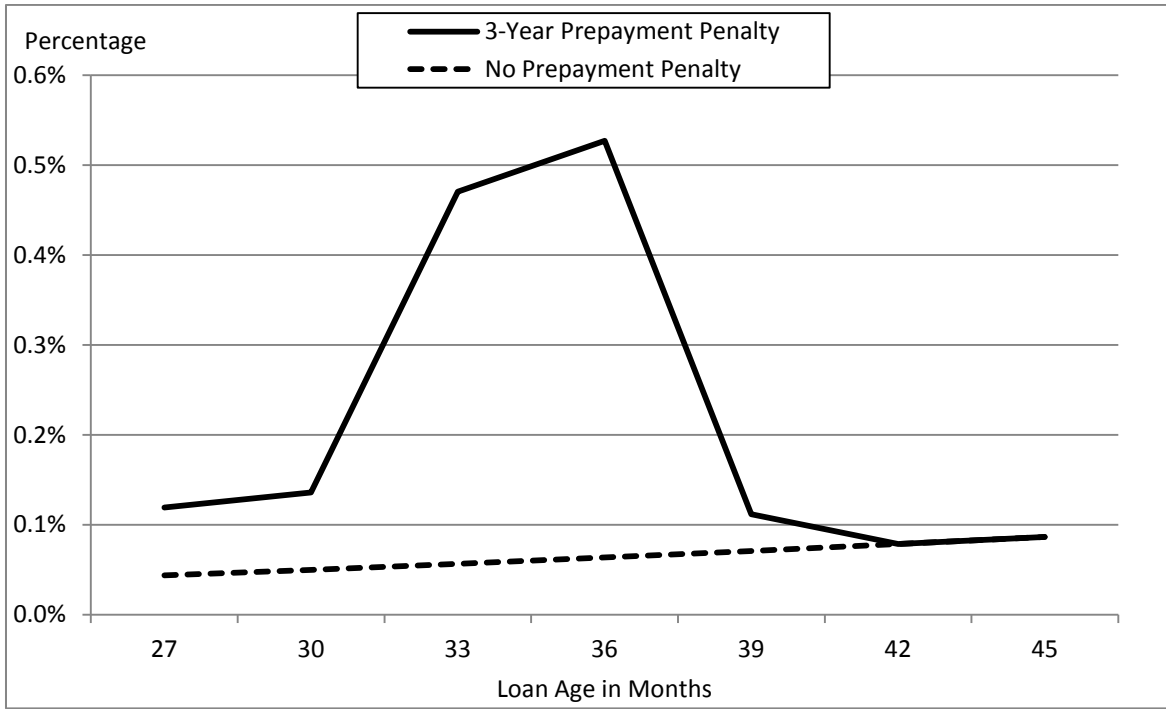
The estimated average probability across all observations at the loan age indicated for a mortgage with a 10-year balloon payment due and another without the balloon payment due using the "Prepayment Timing" specification.

Figure 11: 10-Year Balloon Paid Off Probabilities



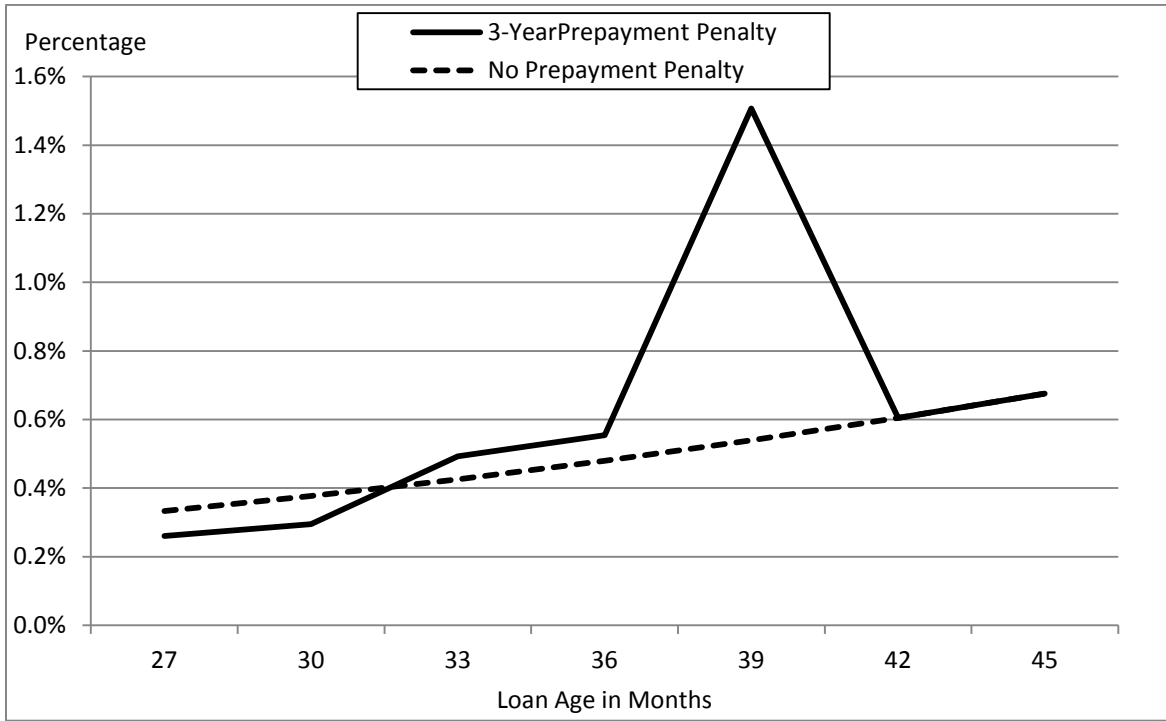
The estimated average probability across all observations at the loan age indicated for a mortgage with a 10-year balloon payment due and another without the balloon payment due using the "Prepayment Timing" specification.

**Figure 12: 3-Year Prepayment Penalty Default Probabilities**



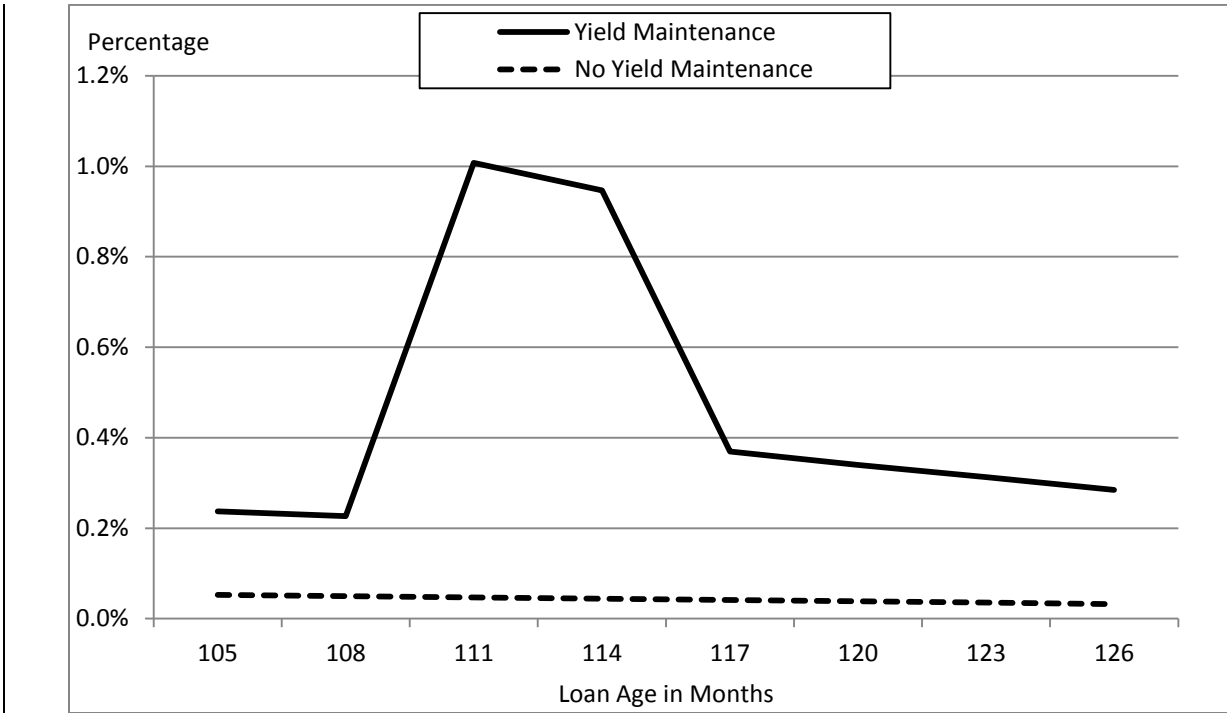
The estimated average probability across all observations at the loan age indicated for a mortgage with a 3-year prepayment penalty and another otherwise identical mortgage without a 3-year prepayment penalty. The “Prepayment Timing” specification is used.

**Figure 13: 3-Year Prepayment Penalty Paid Off Probabilities**



The estimated average probability across all observations at the loan age indicated for a mortgage with a 3-year prepayment penalty and another otherwise identical mortgage without a 3-year prepayment penalty. The “Prepayment Timing” specification is used.

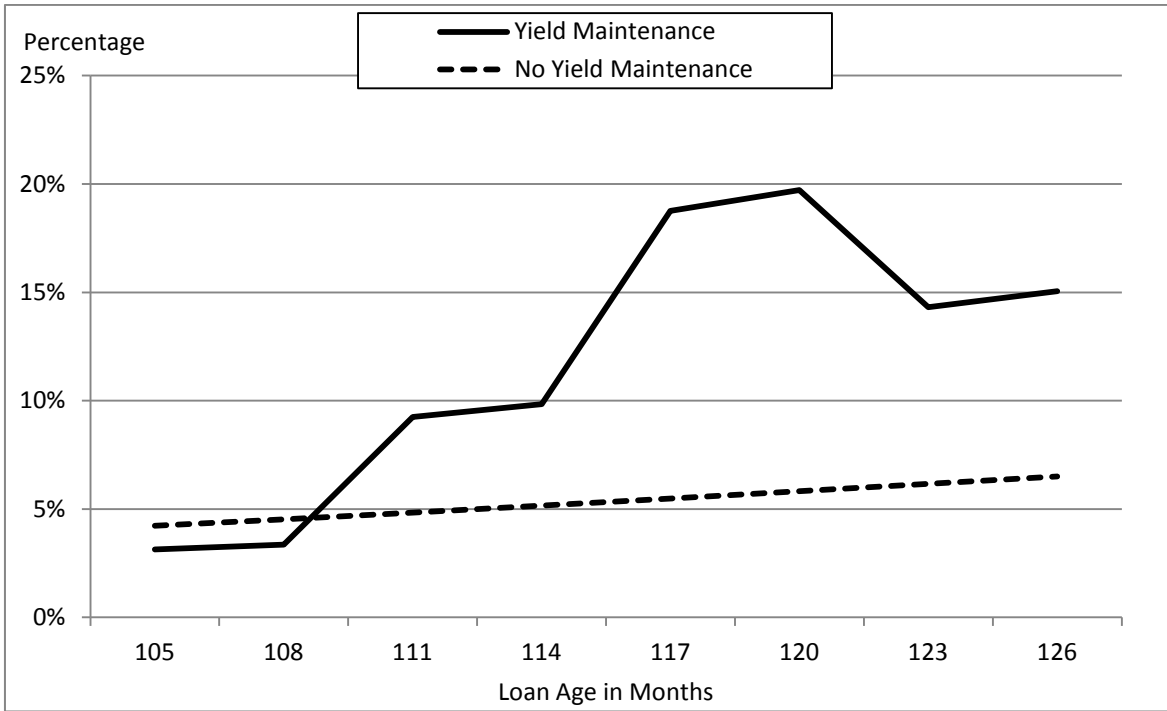
**Figure 14: 9.5 Year Yield Maintenance Expires and Default Probabilities**



The estimated average probability across all observations at the loan age indicated for a mortgage with a yield maintenance feature and another otherwise identical mortgage without the yield maintenance feature. The "Prepayment Timing" specification is used.

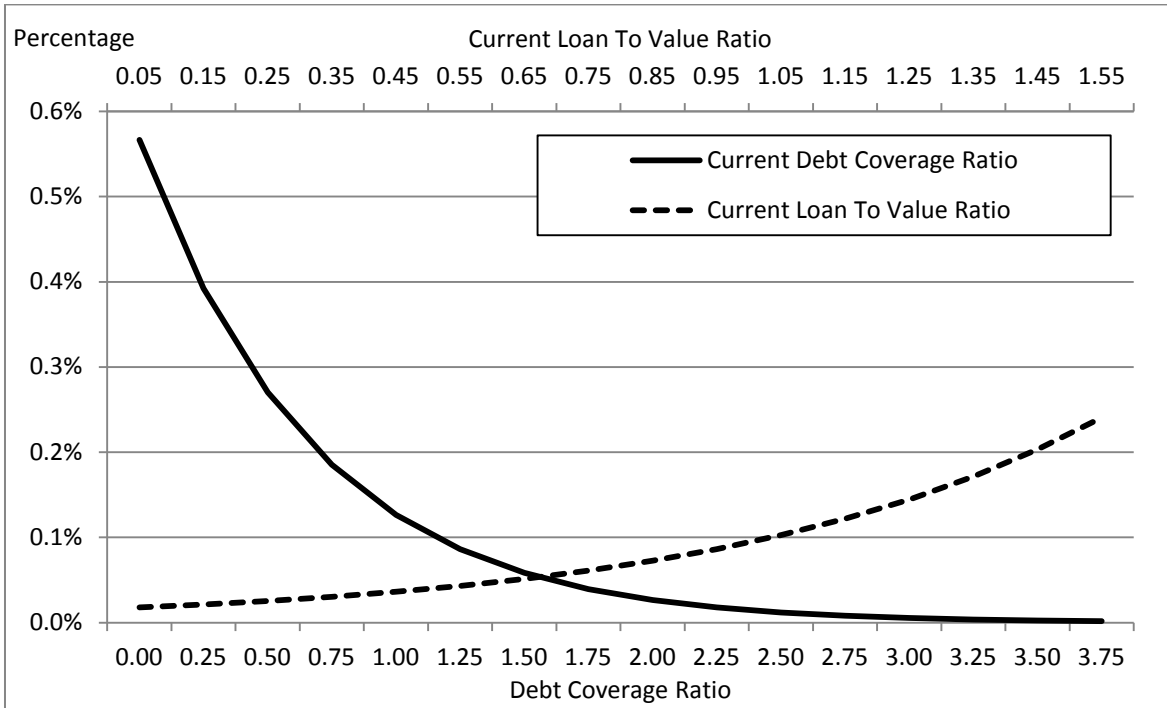


**Figure 15: 9.5 Year Yield Maintenance Expires and Paid Off Probabilities**



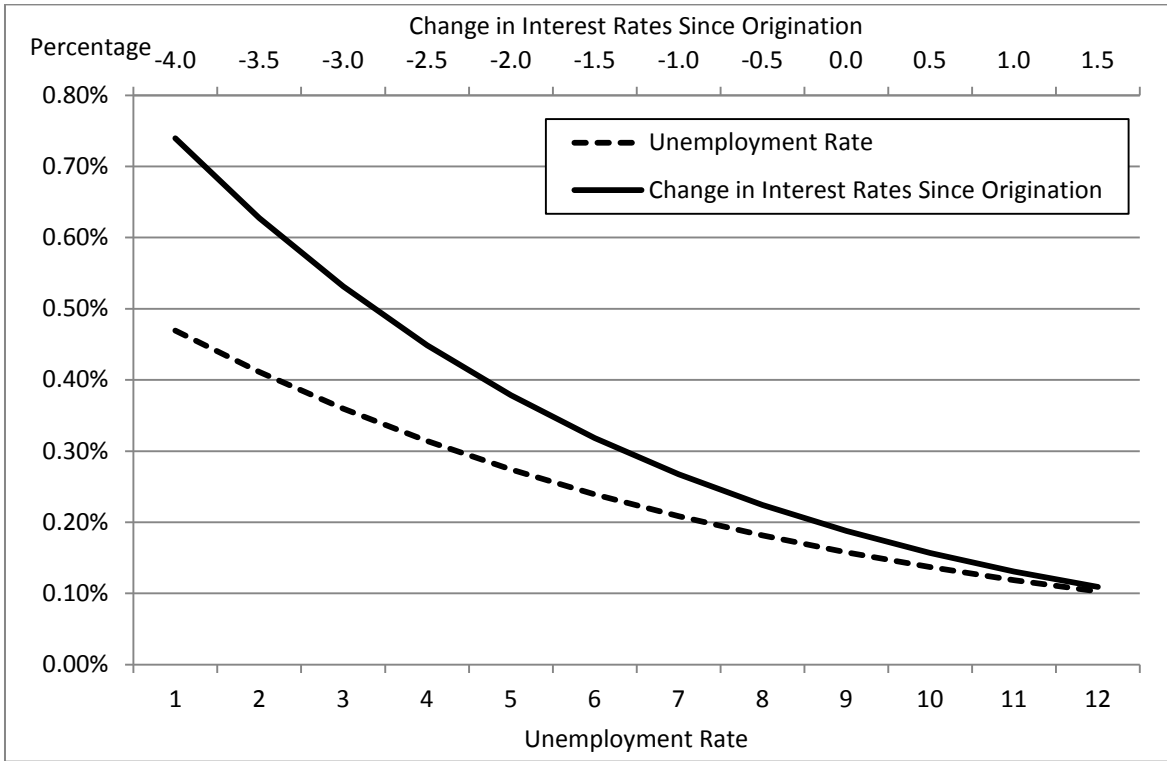
The estimated average probability across all observations at the loan age indicated for a mortgage with a yield maintenance feature and another otherwise identical mortgage without the yield maintenance feature. The "Prepayment Timing" specification is used.

**Figure 16: Current Loan to Value Ratio, Current Debt Coverage Ratio, and Default**



The estimated average probability across all observations at the current debt cover ratio (*dcr\_noi*) or the current loan to value ratio (*cltv*) indicated. The “Endogeneity Test” specification is used.

**Figure 17: Interest Rates, Unemployment Rates and Paid Off**



The estimated average probability across all observations at the county unemployment rate (*urate*) or change in interest rates ( $\Delta irate$ ) indicated. The "Market Conditions and Performance Ratios" specification is used.

**Table 1: Data Dictionary - Basic and Ratios**

| <b>Variables</b> | <b>Definition</b>  |
|------------------|--|
| <i>Basic</i>     |  |
| loanage          | Age of loan in months.   |
| loanage2         | Age of loan squared.   |
| upb_orig         | Loan amount or unpaid balance in 100,000 dollars at origination.   |
| refi             | 1 if the loan is a refinance loan. 0 if the loan is for the purchase of the property.  |
| term             | Term of loan at origination in years.  |
| recourse         | 1 if the lender/investor has recourse to other assets beyond the property if the loan defaults. 0 if the lender/investor does not have access to any assets beyond the property.   |
| t_spread         | The difference or spread between the risk free interest rate (10 year constant maturity treasury) at origination and the interest rate on the mortgage at origination.   |
| arm              | 1 for adjustable rate mortgage. 0 for a fixed rate mortgage.   |
| io               | 1 for an interest only mortgage. 0 for an amortizing mortgage.   |
| <i>Ratios</i>    |  |
| dcr_acq          | The debt coverage ratio (annual net operating income / annual debt service) at origination   |
| dcr_noi          | The debt coverage ratio (annual debt service / net operating income) in the current year.  |
| ltv_orig         | The loan to value (ltv = loan amount / property value) at origination  |
| cltv             | The ratio of the current (quarterly) loan amount to the current property value. The current loan amount is reported directly in the servicing data. The current property value is derived using data from Costar. The value of the property at origination is updated to the current quarter using the change in the observed price per square foot of multifamily property at the state level from transactions reported by Costar. In some states it was necessary to aggregate up to the Census Division level due to very few observed transactions. |

**Table 2: Data Dictionary - Market and Prepayment**

|               |  |
|---------------|--|
| <i>Market</i> |  |
| urate         | The state of the property's location unemployment rate state expresses in percentages. This data is collected from the Bureau of |

|                   |   |
|-------------------|---|
|                   | Labor and Statistics.   |
| $\Delta$ rent     | Change in prevailing average rent for multifamily property over the prior quarter. Rent is measured at the county level as the Fair Market Rent reported by the Department of Housing and Urban Development.  |
| $\Delta$ cap      | The change in prevailing cap (capitalization) rates (net operating income / property value) over the prior quarter. Cap rate data is generated by averaging observed multifamily transaction cap rates to the state level. In some states it was necessary to aggregate up to the Census Division level. This data is collected from Costar. For each observed transaction with a capitalization rate we cleaned the data to include only those that were under \$5 million dollars and represented single property transactions. |
| $\Delta$ irate    | The change in prevailing interest rates over the prior quarter. The interest rate series used is the 50 <sup>th</sup> percentile of the observed “small loan” originated interest rate for fixed rate loans.  |
| <i>Prepayment</i> |   |
| ym                | 1 in all time periods before a yield maintenance provision expires. 0 in time periods when a yield maintenance provision is not in effect.  |
| ym_t01            | 1 if the yield maintenance provision ends in the current quarter or the next quarter. 0 otherwise.  |
| ym_tn2            | 1 if the yield maintenance provision has already ended in the last two quarters. 0 otherwise.   |
| ym_tn3            | 1 if the yield maintenance provision has already ended in the three or more quarters ago. 0 otherwise.  |
| paypen            | 1 in all time periods before a prepayment penalty provision expires. 0 in time periods when a prepayment penalty provision is not in effect.  |
| paypen_t01        | 1 if the prepayment penalty provision ends in the current quarter or the next quarter. 0 otherwise.   |
| paypen_tn         | 1 if the prepayment penalty provision has already ended during the last quarter. 0 otherwise.   |
| balloon           | 1 if the mortgage is not fully amortizing. 0 if the mortgage is fully amortizing.   |
| balloon_t0        | 1 if the mortgage is in the last quarter of the loan’s scheduled life and the balloon payment is due. 0 otherwise.  |
| balloon_t1        | 1 if the mortgage’s life ends in the next quarter and the loan is not fully amortizing. 0 otherwise   |

**Table 3: Data Dictionary - Building, Operations, and Product Type**

| <i>Building and Operations</i> |  |
|--------------------------------|--|
| b_units                        | The number of units in the building or buildings that back the mortgage.   |
| year_built                     | The year the building or average year the buildings were built.  |
| egi/pgi                        | The ratio of effective gross income to potential gross income.   |
| oper%                          | The percent of income used by operating expenses.  |
| capex%                         | Capital expenditures as a fraction of effective gross income.  |
| reserves                       | The dollar amount of replacement reserve balance at the start of the year.   |
| <i>Product and Channel</i>     |  |
| mflex                          | 1 if the mortgage was originated/underwritten/acquired in the MFLEX product line. 0 for all other product lines.           |
| pa                             | 1 if the mortgage was originated/underwritten/ acquired in the Prior Approval product line. 0 for all other product lines. |
| mbs                            | 1 if the mortgage was acquired through a mortgage backed security. 0 if the mortgage was acquired through other methods.   |

**Table 4: Summary Statistics**

| Category                      | Variables  | Mean      | Std. Dev   | Min      | Max          |
|-------------------------------|------------|-----------|------------|----------|--------------|
| Basic                         | loanage    | 37.792    | 25.840     | 1        | 201.45       |
|                               | loanage2   | 2,328.75  | 2,715.78   | 1        | 40,898.36    |
|                               | upb_orig   | 17.693    | 9.624      | 0.11     | 50           |
|                               | refi       | 0.662     | 0.473      | 0        | 1            |
|                               | term       | 123.374   | 56.320     | 6        | 360          |
|                               | recourse   | 0.917     | 0.274      | 0        | 1            |
|                               | t_spread   | 2.601     | 0.931      | -0.02    | 4.12         |
|                               | arm        | 3.98E-04  | 0.020      | 0        | 1            |
|                               | io         | 0.109     | 0.312      | 0        | 1            |
| Ratios                        | dcr_acq    | 2.322     | 3.248      | 1        | 85.72        |
|                               | dcr_noi    | 0.492     | 1.591      | -4.51    | 55.01        |
|                               | ltv_orig   | 0.586     | 0.204      | 5.90E-03 | 0.97         |
|                               | cltv       | 0.555     | 0.280      | 0.00     | 2.77         |
| Market                        | urate      | 7.662     | 2.170      | 3.17     | 13.63        |
|                               | Δrent      | 0.028     | 0.035      | -0.18    | 0.73         |
|                               | Δcap       | 0.238     | 0.604      | -2.56    | 3.53         |
|                               | Δirate     | -0.322    | 0.647      | -3.00    | 0.85         |
| Prepayment                    | ym         | 0.901     | 0.257      | 0        | 1            |
|                               | ym_t01     | 0.007     | 0.028      | 0        | 1            |
|                               | ym_tn2     | 0.009     | 0.036      | 0        | 0.67         |
|                               | ym_tn3     | 0.004     | 0.042      | 0        | 1            |
|                               | paypen     | 0.481     | 0.488      | 0        | 1            |
|                               | paypen_t01 | 0.003     | 0.013      | 0        | 0.25         |
|                               | paypen_tn  | 0.003     | 0.013      | 0        | 0.33         |
|                               | balloon    | 0.998     | 0.023      | 0        | 1            |
|                               | balloon_t0 | 0.003     | 0.014      | 0        | 0.33         |
|                               | balloon_t1 | 0.006     | 0.019      | 0        | 0.33         |
| Building<br>and<br>Operations | b_units    | 6.512     | 6.748      | 1        | 148          |
|                               | year_built | 1957      | 29         | 1800     | 2011         |
|                               | egi/pgi    | 1.021     | 0.206      | 0.53     | 15.46        |
|                               | oper%      | 53.536    | 15.852     | 12.86    | 172.41       |
|                               | capex%     | 0.030     | 0.147      | 0        | 10.71        |
|                               | reserves   | 27,552.41 | 126,801.40 | 0        | 2,186,169.00 |
| Product<br>and<br>Channel     | mflex      | 0.130     | 0.336      | 0        | 1            |
|                               | pa         | 0.239     | 0.426      | 0        | 1            |
|                               | mbs        | 0.451     | 0.498      | 0        | 1            |

**Table 5: Base Model of Early Termination**

| Category                  | Variable     | Default      |                | Prepayment  |                |
|---------------------------|--------------|--------------|----------------|-------------|----------------|
|                           |              | Coefficient  | Standard Error | Coefficient | Standard Error |
| Basic                     | loanage      | 0.062***     | 0.013          | 0.044***    | 0.004          |
|                           | loanage2     | -2.79E-04*** | 1.02E-04       | -4.00E-05   | 2.98E-05       |
|                           | upb_orig     | -0.043***    | 0.011          | -0.001      | 0.003          |
|                           | refi         | -0.032       | 0.264          | -0.152**    | 0.063          |
|                           | term         | -0.016***    | 0.005          | -0.025***   | 0.002          |
|                           | recourse     | 0.267        | 0.440          | 0.053       | 0.151          |
|                           | arm          |              |                | -0.318      | 0.506          |
|                           | io           | 0.659***     | 0.255          | 0.208**     | 0.085          |
| Ratios                    | dcr_acq      | 0.034        | 0.027          | -0.044**    | 0.022          |
|                           | ltv_orig     | 3.164***     | 0.606          | 0.021       | 0.181          |
| Prepayment                | ym           | -0.952***    | 0.287          | -2.149***   | 0.072          |
|                           | paypen       | -0.562       | 0.346          | -1.825***   | 0.099          |
|                           | balloon      | -0.264       | 0.456          | 1.617***    | 0.203          |
| Building                  | b_units      | 0.031**      | 0.015          | 0.032***    | 0.006          |
|                           | year_built   | -0.002       | 0.003          | -0.006***   | 0.001          |
| Product<br>and<br>Channel | mflex        | 1.235***     | 0.450          | -0.061      | 0.247          |
|                           | pa           | 0.489*       | 0.293          | -0.002      | 0.091          |
|                           | mbs          | -0.184       | 0.265          | 0.254***    | 0.071          |
|                           | constant     | -4.877       | 5.563          | 8.427***    | 1.964          |
|                           | Loans        | 10,102       |                |             |                |
|                           | Observations | 166,926      |                |             |                |
|                           | Wald         | 6721.81      |                |             |                |

\*\*\* indicates the coefficient is significant at the 99 percent level, \*\* indicates the coefficient is significant at the 95 percent level, \* indicates the coefficient is significant at the 90 percent level. Due to the very low number of adjustable rate loans that defaulted we exclude it as a covariate in the default portion of the likelihood function. The standard errors are clustered for each loan.



**Table 6: Base Model of Early Termination –  
Predicted Probabilities for Indicator Variables**

| <b>Variable</b> | <b>Paid Off</b> | <b>Default</b> |
|-----------------|-----------------|----------------|
| refi            |                 |                |
| no              | 1.40%           | 0.11%          |
| yes             | 1.23%           | 0.11%          |
| recourse        |                 |                |
| no              | 1.26%           | 0.08%          |
| yes             | 1.31%           | 0.11%          |
| io              |                 |                |
| no              | 1.30%           | 0.10%          |
| yes             | 1.54%           | 0.19%          |
| ym              |                 |                |
| no              | 5.25%           | 0.20%          |
| yes             | 0.74%           | 0.09%          |
| paypen          |                 |                |
| 0               | 1.94%           | 0.13%          |
| 1               | 0.36%           | 0.08%          |
| balloon         |                 |                |
| 0               | 0.32%           | 0.14%          |
| 1               | 1.37%           | 0.11%          |
| mflex           |                 |                |
| 0               | 1.31%           | 0.10%          |
| 1               | 1.24%           | 0.35%          |
| pa              |                 |                |
| 0               | 1.32%           | 0.08%          |
| 1               | 1.31%           | 0.13%          |
| mbs             |                 |                |
| 0               | 1.21%           | 0.11%          |
| 1               | 1.50%           | 0.09%          |

The average estimated probability (for all observations) is reported while holding the value of the variable of concern at indicated level.

**Table 7: Prepayment Timing**

| Category       | Variable     | Default      |                | Prepayment   |                |
|----------------|--------------|--------------|----------------|--------------|----------------|
|                |              | Coefficient  | Standard Error | Coefficient  | Standard Error |
| Basic          | loanage      | 0.072***     | 0.012          | 0.053***     | 0.004          |
|                | loanage2     | -3.99E-04*** | 9.64E-05       | -1.29E-04*** | 2.56E-05       |
|                | upb_orig     | -0.035***    | 0.011          | 0.001        | 0.003          |
|                | refi         | 0.070        | 0.277          | -0.127*      | 0.072          |
|                | term         | -0.005*      | 0.003          | -0.014***    | 0.002          |
|                | recourse     | 0.150        | 0.432          | -0.004       | 0.159          |
|                | arm          |              |                | -0.814*      | 0.465          |
|                | io           | 0.933***     | 0.271          | 0.441***     | 0.084          |
| Ratio          | dcr_acq      | 0.049***     | 0.019          | -0.034       | 0.021          |
|                | ltv_orig     | 3.560***     | 0.608          | 0.163        | 0.197          |
| Prepayment     | ym           | 1.498***     | 0.345          | -0.321**     | 0.147          |
|                | ym_t01       | 1.660**      | 0.661          | 1.076***     | 0.171          |
|                | ym_tn2       | 2.445***     | 0.424          | 1.479***     | 0.163          |
|                | ym_tn3       | 2.349***     | 0.481          | 0.999***     | 0.208          |
|                | paypen       | 1.027***     | 0.254          | -0.243*      | 0.134          |
|                | paypen_t01   | 1.247**      | 0.499          | 0.474**      | 0.206          |
|                | paypen_tn    | 0.611        | 0.442          | 1.151***     | 0.151          |
|                | balloon      | -1.328***    | 0.453          | 0.294        | 0.223          |
|                | balloon_t0   | 4.331***     | 0.355          | 3.075***     | 0.122          |
|                | balloon_t1   | 1.659***     | 0.448          | 1.520***     | 0.107          |
| Building       | b_units      | 0.026**      | 0.014          | 0.030***     | 0.006          |
|                | year_built   | -0.003       | 0.003          | -0.007***    | 0.001          |
| Product<br>and | mflex        | 0.497        | 0.368          | -0.537**     | 0.241          |
|                | pa           | 0.539*       | 0.319          | -0.064       | 0.098          |
| Channel        | mbs          | -0.106       | 0.264          | 0.242***     | 0.077          |
|                | constant     | -5.331       | 5.724          | 7.711***     | 2.220          |
|                | Loans        | 10,102       |                |              |                |
|                | Observations | 166,926      |                |              |                |
|                | Wald         | 7823.96      |                |              |                |

\*\*\* indicates the coefficient is significant at the 99 percent level, \*\* indicates the coefficient is significant at the 95 percent level, \* indicates the coefficient is significant at the 90 percent level. Due to the very low number of adjustable rate loans that defaulted we exclude it as a covariate in the default portion of the likelihood function. The standard errors are clustered for each loan.

**Table 8: Market Conditions & Performance Ratios**

| Category                      | Variable     | Default     |                | Prepayment  |                |
|-------------------------------|--------------|-------------|----------------|-------------|----------------|
|                               |              | Coefficient | Standard Error | Coefficient | Standard Error |
| Basic                         | loanage      | 0.157***    | 0.046          | 0.221***    | 0.042          |
|                               | loanage2     | -0.001***   | 4.59E-04       | -0.001***   | 3.81E-04       |
|                               | upb_orig     | -0.040**    | 0.020          | 0.022***    | 0.008          |
|                               | refi         | -0.011      | 0.352          | -0.449      | 0.276          |
|                               | term         | -0.003      | 0.003          | -0.026***   | 0.010          |
|                               | recourse     | -1.514***   | 0.573          | 1.329       | 1.187          |
|                               | t_spread     | -0.013      | 0.253          | -0.327***   | 0.129          |
|                               | io           | 0.740**     | 0.347          | 0.349*      | 0.186          |
| Ratio                         | dcr_acq      | -0.176      | 0.292          | -0.072      | 0.050          |
|                               | ltv_orig     | 7.745***    | 1.300          | -0.945*     | 0.513          |
| Market                        | urate        | 0.193**     | 0.081          | -0.148***   | 0.056          |
|                               | Δrent        | -5.915      | 3.830          | 1.735       | 2.751          |
|                               | Δcap         | 0.274**     | 0.130          | -0.135      | 0.099          |
|                               | Δirate       | -0.500      | 0.306          | -0.383**    | 0.175          |
| Prepayment                    | ym           | 1.033       | 0.692          | -0.635      | 0.440          |
|                               | ym_t01       |             |                | 1.184**     | 0.539          |
|                               | ym_tn2       |             |                | 1.669***    | 0.517          |
|                               | ym_tn3       |             |                | 1.974***    | 0.606          |
|                               | paypen       | 0.297       | 0.384          | 0.207       | 0.347          |
|                               | paypen_t01   | 2.386***    | 0.776          | -0.454      | 0.651          |
|                               | paypen_tn    | -1.347      | 1.409          | 1.398***    | 0.516          |
|                               | balloon      | -2.285***   | 0.635          | 1.143       | 0.716          |
|                               | balloon_t0   | 4.350***    | 0.855          | 1.124**     | 0.575          |
|                               | balloon_t1   | 1.822       | 0.878          | 0.641       | 0.584          |
| Building<br>and<br>Operations | b_units      | 0.083**     | 0.037          | 0.024       | 0.028          |
|                               | year_built   | -0.020***   | 0.005          | -0.007**    | 0.004          |
|                               | egi/pgi      | -4.723***   | 2.029          | -1.139      | 0.845          |
|                               | oper%        | 0.019***    | 0.005          | 0.009       | 0.006          |
|                               | capex%       | 0.850       | 2.003          | -1.636      | 1.868          |
|                               | reserves     | -7.17E-06   | 5.56E-06       | -8.52E-06   | 6.02E-06       |
| Product<br>and<br>Channel     | mflex        | 0.224       | 0.590          | 0.231       | 0.420          |
|                               | pa           | 0.482       | 0.454          | -0.450      | 0.349          |
|                               | mbs          | 0.947**     | 0.414          | 0.490       | 0.324          |
|                               | constant     | 27.275***   | 10.812         | 5.201       | 7.425          |
|                               | Loans        | 4,984       |                |             |                |
|                               | Observations | 77,408      |                |             |                |
|                               | Wald         | 852.03      |                |             |                |

\*\*\*, \*\*, \* indicate that the coefficient is significant at the 99 percent level, 95 percent level, or 90 percent level. Due to the very low number of adjustable rate loans we exclude it as a covariate. For the same reason the yield maintenance timing variables are excluded from the default portion of the likelihood function. The standard errors are clustered for each loan.

**Table 9: Endogeneity**

| Category                      | Variable     | Default     |                | Prepayment  |                |
|-------------------------------|--------------|-------------|----------------|-------------|----------------|
|                               |              | Coefficient | Standard Error | Coefficient | Standard Error |
| Basic                         | loanage      | 0.184***    | 0.057          | 0.222***    | 0.042          |
|                               | loanage2     | -0.002***   | 0.001          | -0.001***   | 3.82E-04       |
|                               | upb_orig     | -0.052**    | 0.023          | 0.027***    | 0.008          |
|                               | refi         | -0.188      | 0.408          | -0.430      | 0.269          |
|                               | term         | -0.003      | 0.003          | -0.026***   | 0.010          |
|                               | recourse     | -1.241*     | 0.666          | 1.488       | 1.197          |
|                               | t_spread     | -0.031      | 0.224          | -0.323***   | 0.129          |
|                               | io           | 0.750**     | 0.360          | 0.279       | 0.187          |
| Ratio                         | dcr_noi      | -1.600***   | 0.303          | 0.112*      | 0.064          |
|                               | cltv         | 1.768***    | 0.246          | -0.288      | 0.212          |
| Market                        | urate        | 0.045       | 0.092          | -0.147***   | 0.057          |
|                               | Δrent        | -7.065*     | 4.055          | 1.766       | 2.763          |
|                               | Δcap         | 0.154       | 0.148          | -0.116      | 0.102          |
|                               | Δirate       | -0.517*     | 0.296          | -0.389**    | 0.178          |
| Prepayment                    | ym           | 1.104*      | 0.670          | -0.646      | 0.456          |
|                               | ym_t01       |             |                | 1.213**     | 0.556          |
|                               | ym_tn2       |             |                | 1.696***    | 0.532          |
|                               | ym_tn3       |             |                | 1.917***    | 0.671          |
|                               | paypen       | 0.261       | 0.404          | 0.242       | 0.367          |
|                               | paypen_t01   | 2.499***    | 0.793          | -0.521      | 0.659          |
|                               | paypen_tn    | -1.305      | 1.423          | 1.402***    | 0.498          |
|                               | balloon      | -2.383***   | 0.711          | 1.121       | 0.734          |
|                               | balloon_t0   | 4.319***    | 0.802          | 1.133**     | 0.565          |
|                               | balloon_t1   | 1.793*      | 0.954          | 0.658       | 0.587          |
| Building<br>and<br>Operations | b_units      | 0.121***    | 0.045          | 0.008       | 0.029          |
|                               | year_built   | -0.016***   | 0.005          | -0.007**    | 0.004          |
|                               | egi/pgi      | -3.511      | 2.446          | -1.170      | 0.850          |
|                               | oper%        | -0.028***   | 0.011          | 0.012**     | 0.006          |
|                               | capex%       | 0.598       | 2.282          | -1.618      | 1.870          |
|                               | reserves     | -6.97E-06   | 5.12E-06       | -8.46E-06   | 5.94E-06       |
| Product<br>and<br>Channel     | mflex        | -0.282      | 0.564          | 0.230       | 0.422          |
|                               | pa           | 0.458       | 0.488          | -0.416      | 0.342          |
|                               | mbs          | 0.844**     | 0.438          | 0.507       | 0.326          |
|                               | constant     | 28.065***   | 10.394         | 3.528       | 7.436          |
|                               | Loans        | 4,984       |                |             |                |
|                               | Observations | 77,391      |                |             |                |
|                               | Wald         | 898.17      |                |             |                |

\*\*\*, \*\*, \* indicate that the coefficient is significant at the 99 percent level, 95 percent level, or 90 percent level. Due to the very low number of adjustable rate loans we exclude it as a covariate. For the same reason the yield maintenance timing variables are excluded from the default portion of the likelihood function. The standard errors are clustered for each loan.

**Table 10: Prepayment Penalties and Yield Maintenance Interactions with Termination Drivers**

| Outcome  | Variable       | With Yield Maintenance  |                | Without Yield Maintenance  |                |
|----------|----------------|-------------------------|----------------|----------------------------|----------------|
|          |                | Marginal Effect         | Standard Error | Marginal Effect            | Standard Error |
| Default  | dcr_noi        | -0.15%***               | 0.03%          | 0.00%                      | 0.01%          |
|          | cltv           | 0.16%***                | 0.03%          | 0.02%                      | 0.02%          |
|          | $\Delta$ irate | -0.06%**                | 0.03%          | 0.03%**                    | 0.02%          |
| Paid Off | dcr_noi        | 0.04%***                | 0.01%          | -0.06%***                  | 0.05%          |
|          | cltv           | -0.03%***               | 0.04%          | -0.15%***                  | 0.10%          |
|          | $\Delta$ irate | -0.03%                  | 0.04%          | -0.19%**                   | 0.08%          |
|          |                | With Prepayment Penalty |                | Without Prepayment Penalty |                |
|          |                | Marginal Effect         | Standard Error | Marginal Effect            | Standard Error |
| Default  | dcr_noi        | -0.15%                  | 0.04%          | -0.09%***                  | 0.03%          |
|          | cltv           | 0.19%                   | 0.06%          | 0.10%***                   | 0.03%          |
|          | $\Delta$ irate | 0.02%                   | 0.04%          | -0.08%***                  | 0.04%          |
| Paid Off | dcr_noi        | -0.02%                  | 0.03%          | 0.04%***                   | 0.01%          |
|          | cltv           | -0.08%                  | 0.08%          | -0.05%                     | 0.04%          |
|          | $\Delta$ irate | -0.10%**                | 0.05%          | -0.06%                     | 0.05%          |

\*\*\* indicates the coefficient is significant at the 99 percent level, \*\* indicates the coefficient is significant at the 95 percent level, \* indicates the coefficient is significant at the 90 percent level. These marginal effects are the average marginal effects across all observations. The effects are derived from separate specifications where *dcr\_noi* is interacted with *ym* and then with *paypen* for the *dcr\_noi* results. This is repeated for each of the variables tested and their specific interaction with *ym* and *paypen*.